The Number-Average Size Rule: A New Empirical Relationship between Industrial Location and City Size

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Abstract

The spatial intensities of both industries and population are highly uneven across space. Moreover, these intensities differ not only across industries, but also change through time. Nevertheless, we show using Japanese data for metropolitan areas in two time periods that the location intensities of both industries and population are linked by surprisingly simple and persistent patterns. In particular, we identify a strong negative log-linear relation between the number and the average (population) size of metro areas in which a given industry is found. This relation, which we designate as the Number-Average Size (NAS) Rule, is also shown to be intimately connected to both the Rank-Size Rule and Christaller's[5] Hierarchy Principle applied to metropolitan areas. In particular, we show mathematically that in the presence of the Hierarchy Principle (which holds quite well in Japan) this NAS Rule is essentially equivalent to the Rank Size Rule.

Keywords: Agglomeration; Metropolitan area; Hierarchy Principle; Rank-Size Rule; Zipf's Law; Number-Average Size Rule

JEL Classification: R11, R12

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1 Introduction

The spatial distributions of both industries and population are typically very lumpy, since most economic activities tend to be concentrated in space. For the case of Japan, studied in this paper, more than 80% of the 1999 workforce was located in metro areas, and indeed more than 60% was concentrated in "densely inhabited districts (DID)"[1] occupying only 3% of the available land.[2] Even among these metro areas, the distribution of industries and population is far from uniform, and in addition, the lumpiness of industries varies across sectors. Of the 125 three-digit manufacturing industries in Japan, 38 [resp., 16] have positive employment in less than 50% [resp., 25%] of the metro areas, while 34 are ubiquitous, having positive employment in more than 90% of the metro areas.[3] Moreover, certain metro areas are seen to attract a disproportionately large number of industries, leading to great variation in industrial diversity among metro areas. The most diverse is Tokyo, with positive employment in 122 industries out of the 125, while the least diverse is Iwamizawa, with only 46 industries (and an average diversity of 82 industries across metro areas). In addition, more diverse metro areas tend to have a larger populations.[4] The population distribution among the 113 metro areas in Japan is also very skewed. Of the total population in the metro areas, more than 60% is concentrated in the largest ten areas, and more than 30% in Tokyo alone.

Against this background, the main question addressed in this paper is whether these locational patterns exhibit any strong empirical regularities, or whether they might equally well have happened by chance. Using the Japanese data for 1980/1981 and 1999/2000, our main empirical findings are to show that such regularities do indeed exist. In particular, we show that there is a strong negative log-linear relation between the number and the average (population) size of metro areas in which a given industry is present (shown in Figure 3.1 below). This regularity, which we designate as the *Number-Average Size* (*NAS*) *Rule*, is particularly striking in view of the fact that over this time period industrial locations in Japan have tended to trickle down from large metro areas to a greater number of smaller ones. But in spite of these locational shifts, the NAS Rule has remained stable.

In addition to this new empirical regularity, it is also shown that the industrial location patterns during these two periods are remarkably consistent with Christaller's [5] classic *Hierarchy Principle* of industrial location behavior, which asserts that industries in metro areas of a given size will also be found in all metro areas of larger sizes.[5] Moreover,while many previous case studies have provided empirical support for this principle (e.g., Berry [2], Christaller [5], Dicken and Lloyd [7], Isard [28], Lösch [32], and Marshall [33]), we develop the first formal statistical test of this principle (to our knowledge), and

[1] The DID is defined in the Population Census of Japan (2000) as a geographic area having residential population at least 5000 with population density greater than $4000/km^2$.

[2] In the case of the US, the concentration is more extreme: about 95% of the population lives on 5% of the land.

[3] For futher details on industrial classification and the definition of metro areas, see Section 2.1.

[4] The (Spearman) rank correlation between size of metro areas and their diversity (i.e., the number of industries with positive employment) is greater than 0.9.

[5] A similar but a less strict Hierarchy Principle was suggested by Lösch [32].

establish a strong result for the case of Japan.

Along with its relation to the Hierarchy Principle, the negative log-linearity of the NAS Rule strongly parallels that of the well-known *Rank-Size Rule* (also known as *Zipf's Law*) for city size distributions, which asserts that if cities in a given country are ranked by (population) size, then the rank and size of cities are negatively log-linearly related (with slope usually close to one in absolute value).

The Rank-Size Rule has been noted as early as Auerbach [1] and Zipf [51]. Informal evidence found in various countries is summarized in Rosen and Resnick [45] (see also Carroll [4] for a survey of the empirical literature). While the empirical validity of the Rank-Size Rule is still debatable (e.g., Black and Henderson[3], Gabaix and Ioannides[22] and Soo [49]), it remains as a useful benchmark to describe the size distribution of cities (see Gabaix [21] and Duranton [10] for further discussion). Indeed, the major theoretical finding of this paper is to show that whenever industrial locations are consistent with Christaller's Hierarchy Principle, the NAS Rule for industrial locations and the Rank-Size Rule for metro areas are *essentially equivalent*. In particular, if metro areas follow an appropriate Rank-Size Rule, and industries follow Christaller's Hierarchy Principle, then the NAS Rule must be a necessary consequence. However, our results also show that this equivalence is meaningful only for Rank-Size Rules with negative slope less than one in absolute value.[6] Japan appears to be a clear case in point. But in countries satisfying Rank-Size Rules with negative slopes greater than or equal to one in absolute value, Christaller's Hierarchy Principle precludes the existence of (even asymptotic) log-linear relations of the NAS type. Hence these theoretical results may help to shed new light on the range of possible relationships between industrial location and city size.

In addition, it should be emphasized that previous theoretical investigations of the Hierarchy Principle and the Rank-Size Rule have been largely independent (as detailed further in Section 6.2 below). Hence the present results may also suggest a possible linkage between these two strands of theoretical literature. On the empirical side, it should also be noted the present analysis of Japanese data represents in our view a natural starting point for the study of such linkages. For since the Hierarchy Principle and the Rank-Size Rule are based, respectively, on "monopolar" structures of industrial and population distributions, it is natural to start with a monopolar economy such as Japan, where these regularities can be seen in their most basic forms.[7] In this way, the results of our present study can serve to suggest possible extensions to more complex types of spatial economies (as discussed further in Section 6.2 below).

The remainder of this paper is organized as follows. A description of data used for the analysis and a discussion of the measurement of industrial localization and diversity are given in Section 2. In Section 3, the NAS Rule observed in the Japanese data is presented. In Section 4, a formal test of the Hierarchy

[6] It is to be noted that here we mean the coefficient of the regression of ln(size) on ln(rank), not the more commonly used power law exponent, which is inverse of our case.

[7] For the case of Japan, it is clear that Tokyo is the single dominant city for both industrial and population hierarchies.

Principle is constructed, and the principle is shown to hold with high significance in the system of metro areas in Japan. In Section 5, a simple theoretical model is developed to establish the link between the Rank-Size Rule and NAS Rule via the Hierarchy Principle. Finally we conclude in Section 6, with a number of remarks on both policy and theoretical implications of our findings, as well as on possible extensions and refinements of the present study.

2 Data and measurement

2.1 Population and industrial data

In this paper, we look at the location patterns of industries and of population in 1980/1981 and 1999/2000 (where population data is available for 1980 and 2000, while industrial data for 1981 and 1999). The individual data sources are given below.

Location and population

The geographic unit we consider is the metro area. Here we use the *Metropolitan Employment Area* (MEA) definition of metro areas developed by Kanemoto and Tokuoka [29]. The MEA is defined to construct metropolitan areas of Japan that are comparable to the Core Based Statistical Area (CBSA) of the US (see Office of Management and Budget [40] for the definition of CBSA). Thus, basic definitions of the MEA follows those of MA (Metropolitan Area) of CBSA. Namely, an MEA consists of a central city and suburban municipalities from which workers commute toward the central city, where the central city consists of a central municipality and possibly some multiple subcentral municipalities (defined below).

To derive MEAs in our application below, it is convenient to formalize the following "suburban" relations suggested by Kanemoto and Tokuoka [29]. If the set of all municipalities in the area of interest is designated as the *municipality set*, $\mathbf{M}_0 \equiv \{M_1, \ldots, M_n\}$, then we begin by defining suburban relations among the elements of any *partition*, $\mathbb{M} \equiv \{\mathbf{M}_1, \ldots, \mathbf{M}_m\}$, of $\mathbf{M}_0$ [i.e., any collection of municipality subsets, $\mathbf{M}_i = \{M_{i1}, \ldots, M_{in_i}\} \subseteq \mathbf{M}_0$, with $\bigcup_{i=1}^m \mathbf{M}_i = \mathbf{M}_0$ and $\mathbf{M}_i \cap \mathbf{M}_j = \varnothing$ for all distinct $i, j = 1, \ldots, m$]. For any $\mathbf{M} \in \mathbb{M}$, the *direct suburb*, $S_1(\mathbf{M} \mid \mathbb{M})$, of $\mathbf{M}$ in $\mathbb{M}$ is defined to be the set of all $\mathbf{M}' \in \mathbb{M} - \{\mathbf{M}\}$, for which the largest share (and at least 10%) of all workers in $\mathbf{M}'$ commute to $\mathbf{M}$.[8] Similarly, the *indirect suburbs* of $\mathbf{M}$ can be defined recursively as follows. For each $k \geq 2$, the *k-th suburb*, $S_k(\mathbf{M} \mid \mathbb{M})$, of $\mathbf{M}$ in $\mathbb{M}$ is the direct suburb of the (k-1)-*th* suburb of $\mathbf{M}$ in $\mathbb{M}$ i.e., $S_k(\mathbf{M} \mid \mathbb{M}) = S_1[S_{k-1}(\mathbf{M} \mid \mathbb{M})]$. The relevant *suburb*, $S(\mathbf{M} \mid \mathbb{M})$, of $\mathbf{M}$ in $\mathbb{M}$ is then defined to be the set of all its direct and indirect suburbs, i.e., $S(\mathbf{M} \mid \mathbb{M}) = \bigcup_k S_k(\mathbf{M} \mid \mathbb{M})$.

Using this relation on partitions, we now "grow" an appropriate set of MEAs by means of an iterative procedure starting with the *basic partition*, $\mathbb{M}_0 = \{\{M_1\}, .., \{M_n\}\}$, containing all singleton sets of

[8] More precisely, if $\sigma(\mathbf{M} \mid \mathbf{M}')$ denotes the *share* of all workers in $\mathbf{M}'$ commuting to $\mathbf{M}$, then $\mathbf{M}' \in S_1(\mathbf{M} \mid \mathbb{M})$ iff (i) $\sigma(\mathbf{M} \mid \mathbf{M}') \geq 0.10$, and (ii) $\sigma(\mathbf{M} \mid \mathbf{M}') \geq \sigma(\mathbf{M}'' \mid \mathbf{M}')$ for all $\mathbf{M}'' \in \mathbb{M} - \{\mathbf{M}\}$. For simplicity we ignore the possibility of ties in (ii).

municipalities in $\mathbf{M}_0$. We first identify the "biggest" elements of $\mathbf{M}_0$. A municipality, $M \in \mathbf{M}_0$ is said to be *large* if M contains a DID with population at least 50,000 and with the day-time versus night-time population ratio exceeding 1.[9] A large municipality, $M_{\mathbf{C}}$, is then said to be a *central municipality* whenever it is *not* in the suburb of any other municipality in $\mathbb{M}_0$, i.e., whenever $M_{\mathbf{C}} \notin S(\{M\} \mid \mathbb{M}_0)$ for any $M \in \mathbf{M}_0 - \{M_{\mathbf{C}}\}$. Each central municipality, $M_{\mathbf{C}}$, is then taken as the "seed" for a larger set of municipalities designated as the *central city*, $\mathbf{C}$, generated by $M_{\mathbf{C}}$. To generate $\mathbf{C}$ from $M_{\mathbf{C}}$, we start by forming a (possibly) larger set of municipalities, $\mathbf{C}_1$, consisting of $M_{\mathbf{C}}$ together with all municipalities $M \in S(\{M_{\mathbf{C}}\} \mid \mathbb{M}_0)$ having DID populations of at least 100,000 or at least one third that of $M_{\mathbf{C}}$. Such municipalities are designated as *subcentral municipalities* for $M_{\mathbf{C}}$ in $\mathbb{M}_0$. This collection of sets, $\mathbf{C}_1$, for each $M_{\mathbf{C}}$ together with all other singleton municipalities $\{M\}$ then forms a new partition, $\mathbb{M}_1$, of $\mathbf{M}_0$ which is possibly "lumpier" than $\mathbb{M}_0$. For each $k \geq 2$, these sets and their associated partition are in turn modified iteratively to form sets, $\mathbf{C}_k$, with associated partitions, $\mathbb{M}_k$, where each $\mathbf{C}_k$ now includes $M_{\mathbf{C}}$ together with all *subcentral municipalities* for $M_{\mathbf{C}}$ in $\mathbb{M}_{k-1}$, i.e., all municipalities $M \in S(\mathbf{C}_{k-1} \mid \mathbb{M}_{k-1})$ having DID populations of at least 100,000 or at least one third that of $M_{\mathbf{C}}$. This process continues until there are no more modifications to be made, i.e., until $\mathbb{M}_k = \mathbb{M}_{k-1}$.[10] Finally, to complete the process, let $\mathbb{M}$ $(= \mathbb{M}_k)$ denote the *final partition* of $\mathbf{M}_0$, and for each (*final*) *central city*, $\mathbf{C} \in \mathbb{M}$, let the relevant MEA be defined by $\mathbf{C}$ together with its suburb, $S(\mathbf{C} \mid \mathbb{M})$, in $\mathbb{M}$, i.e., $MEA_{\mathbf{C}} = \{\mathbf{C}, S(\mathbf{C} \mid \mathbb{M})\}$, where $\mathbf{C}$ is designated at the *central city of* $MEA_{\mathbf{C}}$.[11]

Both the municipality population data and the specific municipalities identified in this study are based on the Population Census of Japan in 1980 and 2000.[12] To make the data comparable between these two years, municipality boundaries in 1980 were converted (basically aggregated) to those in 2000.[13] By applying the above criteria, we then identified 105 MEAs in 1980 and 113 in 2000. The population distribution among MEAs is (as usual) quite skewed: the population size of the largest and smallest MEAs are 31.8 million and 67,814 (26.5 million and 98,878), respectively, in 2000 (1980), with average 918,960 (835,281). MEAs account for 82% (75%), 84%(74%) and 33%(27%) of the national population,

[9] There is an exception applied to 13 "major" municipalities designated by government ordinance. Each of these municipalities consists of multiple highly urbanized wards, and hence is considered to be a *major* municipality if one of its wards is *large* in the above sense.

[10] It is worth noting that such a process could in principle lead to infinite cylces, and hence never converge. To see this, observe that for $k > 0$, a subcentral municipality, M, for a central municipality, $M_{\mathbf{C}}$, in M_k might fail to be subcentral municipality for $M_{\mathbf{C}}$ in M_{k+1}. In particular, another central city may expand enough from k to $k+1$ to "capture" the largest commuting share of M in M_{k+1}. Hence some central cities could in priniciple shrink while others expand. However, Tokuoka and Kanemoto report that under reasonable parameter ranges (defining large cities, etc.), the shrinkage of central cities is never observed to occur, and indeed, that the iterative process typically terminates at the second or third iteration

[11] There are of course some differences in diffinition between our MEA and MA in CBSA due to the organization of the data. First, the basic regions in the former are municiparities, while those of the latter are counties, each of which usually consists of a set of municiparities. Second, the central city is identified based on the DID defined by the Japanese census in the former, while it is from *urbanized area* in the latter. Given the size differences in the basic regions, the "appropriate" threshold values in deriving metro areas appear to differ between the two frameworks. But, since in either definition the basic idea is to define a metro area as a multi-centered employment area, the basic results we present below should not be very sensitive to these minor differences.

[12] The municipality here is equivalent to the *shi-cho-son,* which corresponds to city-town-village in the Japanese Census.

[13] Among 3230 counties, MEAs include 1210 (1440) counties in 1980 (2000).

employment and total area, respectively in 2000 (1980).

Industries

The employment data used for the analyses in this paper are classified according to the three-digit Japanese Standard Industry Classification (JSIC) taken from the Establishment and Enterprise Census of Japan in 1981 and 1999, and applied to the respective population data in 1980 and 2000. Since industrial classifications have been disaggregated for most sectors during this 20-year period, we have attempted to reconcile the two classifications by aggregating the 1999 classification. Among the three-digit JSIC-industries, we focus here on manufacturing, services, wholesale and retail, which together include 264 industries.[14]

2.2 Measurement of industrial localization and diversity

To establish the regularities discussed in the introduction, it is essential to identify for each industry the set of MEAs in which that industry operates, here designated as the *industry-choice MEAs* for that industry. To identify the industry-choice MEAs for each industry, an appropriate threshold level of the industry-specific employment within each MEA needs to be set. However, it is shown in Appendix 7.1 that our empirical findings are not very sensitive to the choice of the threshold level. For this reason, in the analysis to follow, we have chosen to include all MEAs with positive employment in an industry as industry-choice MEAs (i.e., we choose a zero threshold level). The number of industry-choice MEAs for a given industry can then be considered as reflecting the *degree of localization* for that industry. Conversely, the number of industries found (i.e., those industries which have positive employment) in a given MEA can be taken to reflect the *industrial diversity* of that area.

It should be noted that these measures differ from the more traditional measures of industrial localization and industrial diversity of metro areas. Our measure of localization puts an emphasis on the *spatial coordination* between industrial and population agglomerations, rather than the *degree of deviation from complete dispersion* as in the most existing localization indices (e.g., Ellison and Glaeser [17]; Krugman [30, Ch.2], Duranton and Overman [12], Mori, Nishikimi and Smith [34]). There is a possible shortcoming of our approach – namely that it ignores the *size* information of the industrial presence in a given MEA. For instance, an MEA with one small plant for a given industry is treated the same as an area containing many large plants in that industry.[15] Nevertheless, we believe that even a minimal presence of an industry in an MEA is a reasonable indicator to judge whether that area is a *viable location* for

[14] In 1981, there were a total of 319 categories (152 manufacturing, 108 services, and 59 wholesale/retail). From this set, those sectors classified as public sectors in either year, or not fittng any of the specific categories above, have been excluded. For those few sectors that are more aggregated in 1999, we follow the 1999-classification. This resulted in a final set of 125 manufacturing, 90 services, and 49 wholesale and retails industries.

[15] In a separate paper (Mori and Smith [36]), a more systematic, information-theoretic approach (which does take into account the size information) is under development in order to identify the location of each industrial agglomeration. This new approach in turn enables us to identify the set of industries agglomerated in each metro area. However, our preliminary analysis shows that our basic results based on the present simpler approach continue to hold true.

that industry. Indeed, the literature suggests that there is a persistence of industrial locations: once an industry has successfully located in an area, the size of this industrial presence will tend to grow over time even in the presence of high turnover rates of establishments.[16] This is also supported by our own data, which indicates that between 1981 and 1999, the number of industries in operation in more than 90% of MEAs has increased, and moreover, that there is no MEA which has completely lost the employment of more than five industries. Thus, if an industry is in operation in an MEA at a given point of time, then it is quite likely to stay there in the future.

As for regional industrial diversity, most prevailing measures are based on *regional employment diversity*. To motivate our choice of a diversity measure, two alternative measures are worth mentioning. One is the *Hirschman-Herfindahl index* (HHI) which is often used as an "inverse" measure of industrial diversity of a region. If s_{ir} is the employment share of industry i in MEA r, then $\mathrm{HHI}_r = \sum_i s_{ir}^2$ (see for example, Henderson [25]; Henderson, Kuncoro and Turner [26]). Notice however that HHI_r is influenced by the difference in labor requirements across industries. For instance, given the number of industries in operation in an MEA, the MEA is evaluated as more diversified if the employment share for each industry is more similar. Thus, for example, an MEA which happens to have an industry with a large labor requirement will look less diversified. But, the industrial diversity of a region should be independent of the specificity in the production technologies for individual industries.

The other alternative is the *relative diversity index* (RDI) by Duranton and Puga [14]. If s_i denotes industry i's employment share in the nation, and if s_{ir} has the same meaning as in HHI_r above, then $\mathrm{RDI}_r = 1/\sum_i |s_{ir} - s_i|$. As with HHI, this index is defined in terms of employment shares, and suffers from the same problems. But, more fundamentally, the notion of the diversity embodied in RDI can sometimes be difficult to interpret. In particular, since the "industrial diversity" of a region is here defined in terms of the similarity between industrial shares in that region and in the nation as a whole, difficulties arise in the typical (Christaller) hierarchical setting where most employment is in low-order ubiquitous industries, and where higher order industries are concentrated in only a few large MEAs. To illustrate this, let n_{ir} denote the employment of industry i in MEA r (and assume that all employment is in MEAs), so that $s_{ir} = n_{ir}/\sum_j n_{jr}$ and $s_i = \sum_r n_{ir}/\sum_j \sum_r n_{jr}$. Then consider the following simple case of four MEAs $(1, 2, 3, 4)$, and two industries, (i, j), with $(n_{i1} = 1, n_{j1} = 1)$ and $(n_{ir} = 1, n_{jr} = 0)$ for all $r = 2, \ldots, 4$. Here the "large" MEA 1 has both industries, i and j, while each of "small" MEAs contains only the low-order industry i. But since it is readily verified that $\mathrm{RDI}_1 = 1.67$ and $\mathrm{RDI}_r = 2.5$ for $r = 2, \ldots, 4$, we see that in this case the MEA with all industries is the least diverse under RDI. The essential reason for this is that since the low-order industry contains most of the national employment,

[16] Henderson et al. [26] argue that one-standard deviation increase in the proportion of 1970 local employment in a specific industry results in 30% increase in 1987 employment controling for urban size, current labor maket conditions, etc. See also Henderson [25] for a related discussion. Dumais et.al [9] report for the case of the US that nearly three quarters of plants existing in 1972 were closed by 1992, and more than half of all manufacturing employees in 1992 did not exist in 1972.

the national employment share of industry i (80%) is much closer to that of MEAs, $2, \ldots, 4$ (100%), than that of MEA 1 (50%).[17]

So for these reasons, it seems to us that neither the HHI measure nor the RDI measure can provide a generally acceptable measure of regional diversity.[18]

3 The Number-Average Size Rule

In this section, we present our main empirical finding, the *NAS Rule* which governs the relation between industrial location and metro area sizes during our study periods, 1980/1981 and 1999/2000. Furthermore, we construct a statistical test to verify that this strong empirical regularity cannot be attributed to chance alone.

3.1 Evidence

3.1.1 The Number-Average Size Rule

Figure 3.1 shows the relationship between the number and average (population) size of industry-choice MEAs for each three-digit industry in 1980/1981 and 1999/2000. Except for three outliers in 1999/2000,[19]

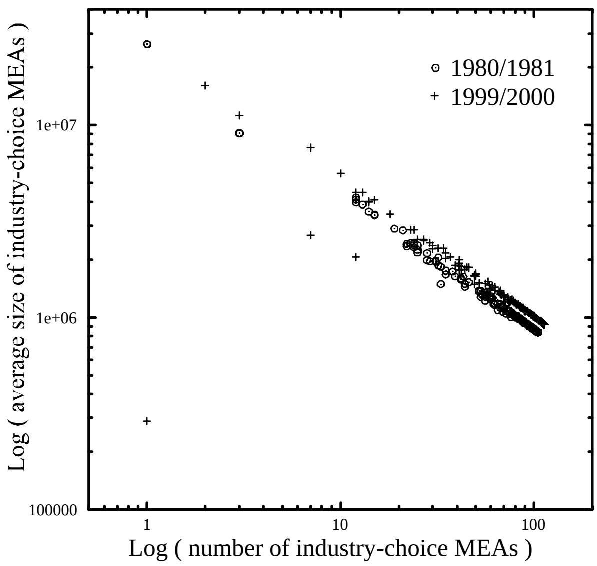

Figure 3.1: Size and number of industry-choice MEAs

[17] In fact, according to Duranton and Puga [14], the three most diverse cities in the US in 1992 in terms of RDI were Cincinnati, Oakland and Atlanta, but not New York, Los Angels and Chicago. Buffalo was listed as the sixth most diverse city, and more diverse than New York. We suspect that our example here illustrates the basic reason for their rather surprising diversity-rankings of US cities.

[18] Refer also to footnote 38.

[19] One of the three outliers is coke manufacturing (JSIC213), while the other two are arms-related industries: small arms manufacturing (JSIC331) and small bullet manufacturing (JSIC333). The location of Coke manufacturing is restricted to the major industrial districts near major sea ports, as it must import inputs nearly completely from overseas and the production is subject to large scale economies. Thus, the relative location to other industries and population may be less important. The latter two outliers may be attributable to the fact that while arms-related industries are technically in the private sector, location decisions in these industries are heavily influenced by government policies in Japan.

the relationship is clear: average size ($\overline{\text{SIZE}}$) is strongly log-linearly related to the number of industry-choice MEAs (#MEA). Ordinary least squares estimation (OLS) gives the following result,

$$1980/1981 : \log(\overline{\text{SIZE}}) = \underset{(0.005193)}{7.376}^{**} - \underset{(0.002750)}{0.7204}^{**} \log(\#\text{MEA}), \quad R^2 = 0.9962, \tag{3.1}$$

$$1999/2000 : \log(\overline{\text{SIZE}}) = \underset{(0.004232)}{7.427}^{**} - \underset{(0.002198)}{0.7124}^{**} \log(\#\text{MEA}), \quad R^2 = 0.9975, \tag{3.2}$$

where the values in the parentheses are standard errors, and ** indicates the coefficient to be significant at 1% level. In fact it should be clear by inspection that the actual significance levels are off the chart (with P-values virtually zero).[20]

Moreover, this relationship is also seen to remain quite stable over time. By pooling the data for both periods and using time dummies, one can apply standard F-tests (Chow tests) to evaluate coefficient shifts. The results of this analysis show that only the intercepts are significantly different (at 5% level) between the two periods, while the slope, i.e., the elasticity between the number and average size of industry-choice MEAs, has been invariant. However the intercept change does reflect a significant effect, namely a differential shift between the numbers of industry-choice MEAs and the average size of those MEAs. While both MEA sizes and numbers of industry-choice MEAs have changed, they have done so in a manner which leaves their elasticity *invariant*. In other words, the percent change in average industry-choice MEA sizes relative to percent change in numbers of industry-choice MEAs has remained essentially constant (with a 1% increase in the number of MEAs chosen by an industry corresponding approximately to an expected decrease of 0.7% in the average sizes of these MEAs). As a parallel to the well-known *Rank-Size Rule* for city size distributions, we designate this new relation as the *Number-Average Size* (*NAS*) *Rule* for industrial location patterns.

But, it is fair to say that such an invariance could in principle be accounted for by simple proportional growth phenomena, such as a constant percent increase in the number of industry-choice MEAs across sectors, or a constant percent increase in average industry-choice MEA sizes across sectors. However, we will see below that it is not at all the case. Also, it turned out that the regularity is distinctively strong when metro areas are chosen as the regional units (as in the present case).[21]

In addition, it should be mentioned that tobacco manufacturing has been excluded from the analysis, because it was classified in the public sector in 1981. By 1999 tobacco was privatized. Hence it is of interest to note that this industry is also an outlier in 1999, and that this again may be attributable to government policies before privatization.

[20] As pointed out by Gabaix and Ioannides [22, Sec. 2.2.1], the standard errors in these regressions may be under-estimated. In order to obtain approximate true standard error, they suggest a Monte-Carlo simulation by drawing the same number of city-size samples as in the actual data from the corresponding power-law distribution. However, their correction of the standard deviation does not seem to apply in our case. It should be kept in mind that the actual city-size distribution need not be a realization from a random sampling from a given power-law distribution. Rather, it is just so happened that the resulting size distribution exhibits an approximate power law, where the underlying mechanism could even be completely deterministic. Given an $R^2 > 0.99$, this relation is so strong that no reasonable estimates of standard errors could make much of a difference in any case.

[21] Refer to Appendix 7.1 for the robustness of the NAS Rule under alternative threshold size for industry-choice MEAs.

3.1.2 Trickling-down phenomenon

Our data indicate that industries have on average tended to disperse their locations to a greater number of smaller metro areas between 1980/1981 and 1999/2000. On the one hand, the average increase in the number of industry-choice MEAs was 12.4%, while the total number of MEAs only increased by 7.6%. But on the other hand, while the average increase in population size of MEAs was 16.7%, the average increase in size among industry-choice MEAs was only 10.4%. Evidently, there has been a *trickling* down of industries from larger MEAs to a greater number of smaller MEAs, tending to lessen the effect of population growth. This trickling down phenomenon can also be confirmed by comparing the changes in the number and average size of industry-choice MEAs across individual industries. In particular, the Spearman's rank correlation between the rates of increase in the number and average size of industry-choice MEAs from 1980/1981 to 1999/2000 is -0.85. This suggests that those industries concentrating their locations in fewer MEAs tend to move from larger to smaller MEAs. Similarly, those industries expanding to larger numbers of MEAs tend to be moving in the opposite direction. More generally, the NAS Rule suggests that this trickling down effect may indeed be highly structured.[22]

While the industries trickle down on average, there is wide variation in the pattern of locational change among individual industries. Figure 3.2 shows the change in the number (Diagram, a) and average size (Diagram, b) of industry-choice MEAs for each industry. In each diagram, the values in 1980/1981 are indicated on the horizontal axis. Diagram (a) shows the change in the number of industry-choice MEAs between 1980/1981 and 1999/2000, which varies widely from -23 for blacksmith services (JSIC801) to 86 for miscellaneous merchandise wholesale (JSIC-S481) with mean = 7.26 and standard deviation = 10.04.[23] In Diagram (b), the average sizes of industry-choice MEAs in 1999/2000 relative to that in 1980/1981 are plotted, which also vary widely from 0.032 for small bullet manufacturing (JSIC333) to 3.80 for fur manufacturing (JSIC248) with mean = 1.09 and standard deviation = 0.240.

The change in the size and industrial diversity of an individual MEA also varies widely across MEAs as shown in Figure 3.3. As in Figure 3.2(b), the value in 1999/2000 (relative to that in 1980/1981) is plotted against the value in 1980/1981 in each diagram. The relative size of an MEA in 1999/2000

[22] The diversification of smaller MEAs seems to imply convergence of industrial structure among all MEAs. However, larger MEAs may also be increasing their diversification, and thus preserving the relative diversity among MEAs. More specifically, under a fixed industrial classification system, it is not possible to capture the formation of new industries, which are more likely to occur in large MEAs. This is well exemplified by computer industries during the 80s and information technology (IT) industries (such as internet-related industries) during the 90s. Among those software, information processing and internet related IT industries found in the Yellow Pages in the year 2000, 46.7%, 10.7% and 4.5% are located in the three largest MEAs, Tokyo, Osaka and Nagoya, respectively. [Data source: Ministry of Land, Infrastracture and Transport of Japan (http://www.mlit.go.jp/).] These IT industries are mostly classfied as information services (JSIC821) which includes wide variety of low-tech traditonal data processing services. Consequently, they appear as a ubiquitous industry found in all MEAs.

[23] It is to be noted that in this diagram, the vertical axis is the absolute change between the two time periods, rather than relative value in 1999/2000 as in other diagrams in Figures 3.2 and 3.3. Since there are several industries with very small numbers of industry-choice MEAs, even small absolute changes can yield enormous percenta change. Thus, if the relative value is used, its statistical variation is highly affected by the size of the base-year value (i.e., the variation deminishes as the base-year value increases), which yields a rather misleading plot.

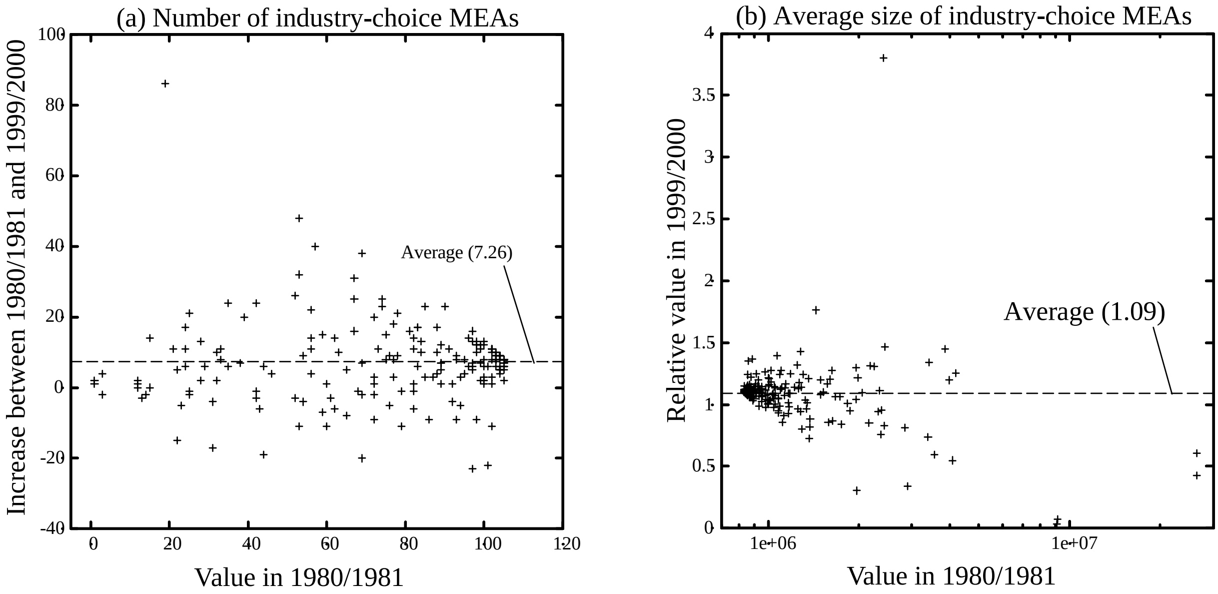

Figure 3.2: Change in the industrial location

(Diagram, a) ranges from 0.471 in Hekinan (23209) to 2.87 in Tsukuba (8220) with mean = 1.17 and standard deviation = 0.272 (where the numbers in the parentheses indicate the jurisdiction code of the central business districts of MEAs). The relative industrial diversity of an MEA in 1999/2000 (Diagram, b) ranges from 0.970 in Ube (35202) in 1.128 of Sanjo (15204) with mean = 1.03 and standard deviation = 0.034.

Given these differential changes among industries and among MEAs, the persistence of the NAS Rule is truly remarkable.[24]

3.1.3 Choice of geographic units

In the analyses above we have focused on the location pattern of industries and population across MEAs. This seems reasonable in view of the fact that MEAs are *economic regions*, essentially embodying the urban economic notion of "cities" (i.e., commuting areas in which individual firms and consumers share a common urban environment). Thus, if population size is a significant determinant of industrial location, then it is natural to expect that MEAs should constitute the appropriate geographic unit.[25] Not surprisingly, if alternative choices of geographic units are used, then the NAS Rule is less evident. Diagrams (a,b) in Figure 3.4 show the 1999/2000 case, where the geographic unit is chosen to be "municipality"and "prefecture," respectively.[26] We can see that the strong pattern of Figure 3.1 is now considerably weaker. This suggests that analyses of industrial location based on inappropriate choices of geographic units can

[24] Duranton[11] has reported similar findings for France and the US. In particular he observes that in these countries the variability of industrial composition across metro areas is much larger than the variability of their population sizes within the national metro-area system.

[25] Duranton and Overman [12] provide evidence for the UK manufacturing that the geographic extent of establishment concentration roughly corresponds to that of an metro area.

[26] Here, the municipalities are "shi-cho-son"s in Japanese, corresponding to "city-town-village"s.

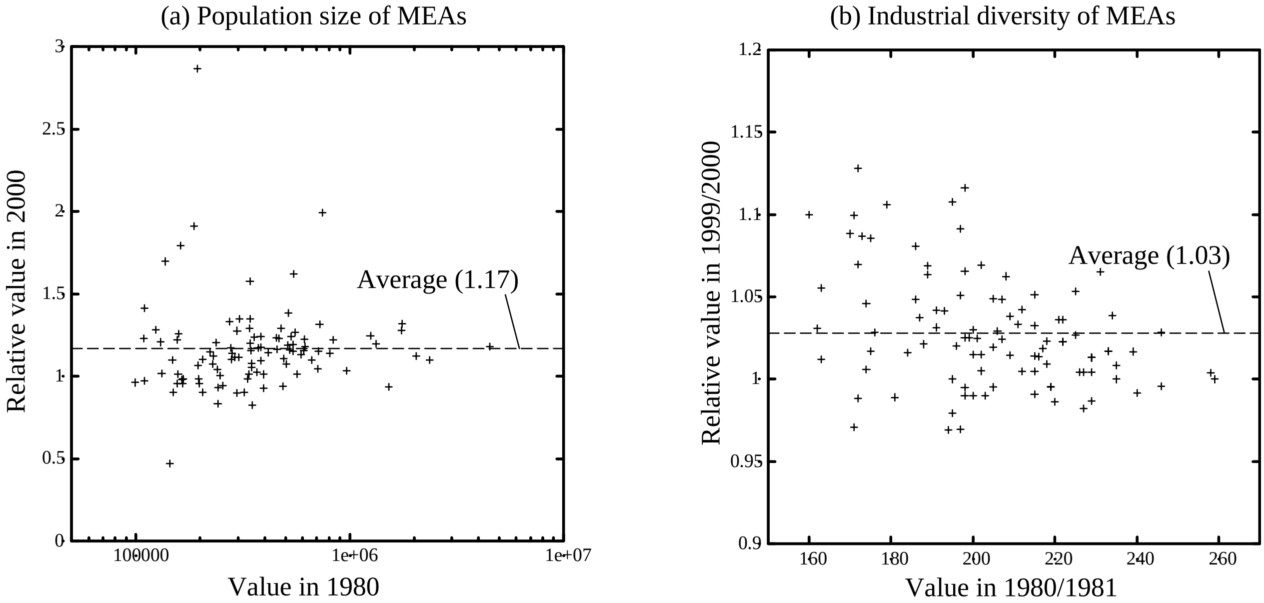

Figure 3.3: Change in the size and industrial diversity of MEAs

be misleading.

3.2 A test of the NAS Rule

To further clarify the significance of this rule, it is of interest to consider what the relationship between the numbers and average sizes of MEAs for each industry would look like if industrial locations were in fact random.[27] To do so, we begin by observing that any process of randomly relocating industries necessarily leads to some ambiguity concerning the resulting "sizes" of MEAs. For example, it makes little sense to relocate a steel plant without considering the employment needed to operate that plant. Hence we consider it more appropriate to relocate employment along with industries. More specifically, we here treat the employment of each industry in an MEA as a single *employment cluster* for that industry, and study the effects of randomly reallocating these clusters among MEAs.[28] This relocation process amounts to randomly "regrowing" the industrial employment structure of each MEA.[29] Ideally, one would also like to relocate the entire population tied to a given industrial cluster (such as the family members of industrial workers). But rather than make arbitrary assumptions as to the sizes of these populations, we choose to focus only on the employment sizes of industrial clusters. Hence in any random reallocation of industrial clusters, we take the resulting *size* of each MEA to be simply the total size of all employment

[27] The negative relationship between the number and average size of industry-choice MEAs can also be tested by comparing the ranking of industries in terms of these two values. Speaman's rank correlations are -0.93 and -0.90 for the cases in 1981 and 1999, respectively. These correlations are both highly significant.

[28] Here we sample without replacement, so that distinct employment clusters are randomly assigned to distinct MEAs.

[29] Note that the size of an MEA (both in terms of population and area) is in reality *not* independent of industrial location. Rather, it is at least in part determined by processes of economic agglomeration. For example, the dramatic growth of Tokyo has been significantly influenced by the decisions of many industries to locate there. Thus, it seems to us to be more natural *not* to take the total size of an MEA as given when industrial location is randomized. While there are of course many alternative randomization schemes, we expect the basic results presented here would be the same.

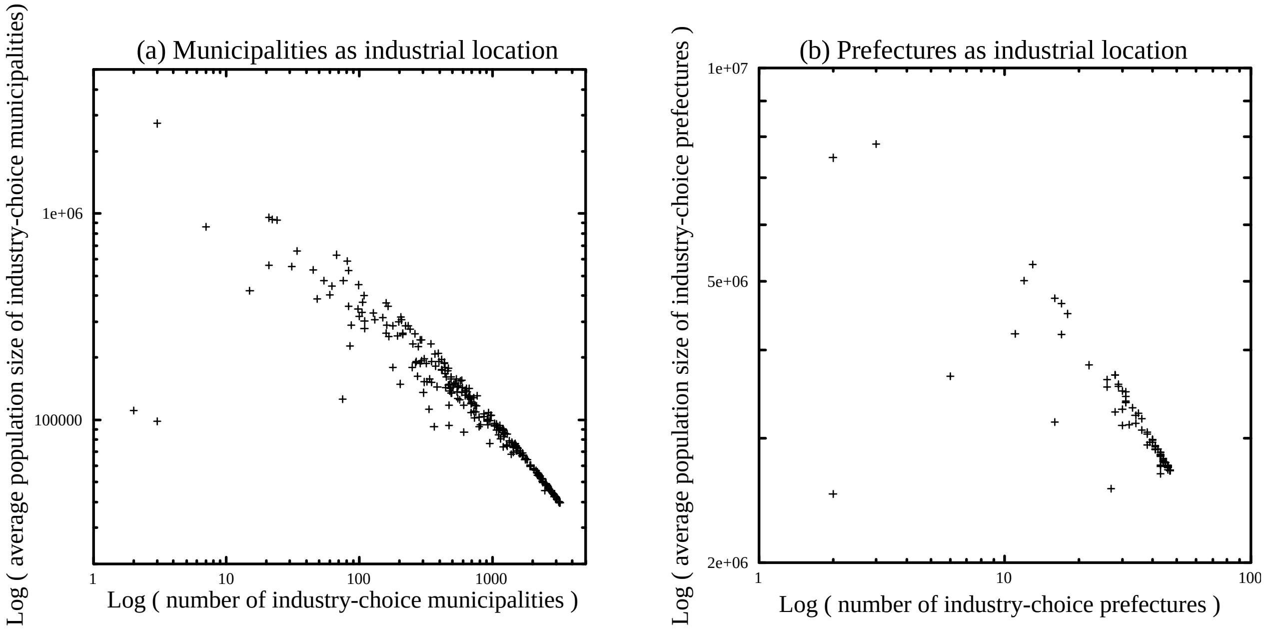

Figure 3.4: Alternative geographical units

clusters assigned to that MEA.

In this context, we now consider the relationship between the number of industry-choice MEAs and the *average employment size* of those MEAs for 1999/2000, as shown by the scatter plot of "+" symbols in Figure 3.5.[30] Notice that this plot closely resembles the same plot in Figure 3.1 using average population sizes of MEAs, and hence that our present restriction to employment sizes evidently has little effect on the NAS Rule itself. To test this actual pattern against random location patterns, we generated 1000 random reallocations of all employment clusters, and calculated the corresponding average employment sizes of the MEAs to which each industry was assigned. The results of these calculations have been summarized in Figure 3.5, by plotting the means (solid horizontal line) together with the 5 and 95 percentile points (broken lines) for the sampling distributions of average MEA sizes for each industry (ordered by their numbers of industry-choice MEAs).[31] These results show quite convincingly that the actual pattern is highly nonrandom, and in particular that the average employment sizes of MEAs for each industry are vastly greater than would be expected by random agglomerations of industries. More formally, if the above sampling distribution is taken to represent the null hypothesis of "purely random industrial agglomerations", then our results provide strong evidence against this null hypothesis.

In fact, the observed average employment sizes not only exceed those of all 1000 samples in every case, but actually approach their *maximum possible levels*, as indicated by the "upper bound" curve in the figure. Here, for each number of industry-choice MEAs, n, the value plotted corresponds to the

[30] Here, the employment of only the selected 264 industries are considered.
[31] The results do not basically change by increasing the number of samples above 1000.

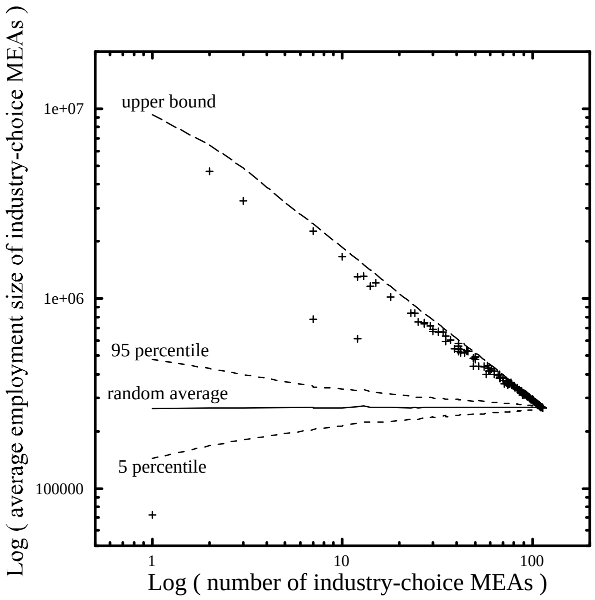

Figure 3.5: A Test of the NAS Rule using 1999/2000 data

actual average size of the largest n MEAs.[32] Hence the actual pattern of average-employment sizes is not only strongly log linear in n, but is seen to be well approximated by its upper bound – which turns out also to be strongly log linear in n. In the next section we introduce Christaller's Hierarchy Principle, which will help to account for this upper-bound approximation. In Section 5 below, we return to the log linearity property of this upper bound, which turns out to be closely related to the log linearity of the more well-known Rank-Size Rule.

4 Hierarchy Principle

Recall from the Introduction that Christaller's Hierarchy Principle asserts that industries found in any given metro area will also be found in all larger metro areas. The basic mechanism underlying this principle is related to one of the most distinctive features of industrial location behavior observed in a domestic economy. Since interregional migration is generally not costly (compared to international one), firms are attracted mainly by the absolute advantage of locations. As a result, certain advantageous location are seen to attract a disproportionately large number of industries, leading to greater variation in industrial diversity among locations. In particular, the set of industries found in a (less advantageous) smaller city is roughly the subset of that found in a (more advantageous) larger one. That is, *more localized industries under the Hierarchy Principle are thus found in larger cities.*[33]

[32] Strictly speaking, this upper bound is relevant only for the actual location pattern (the '+' points) and not for the 1000 random samples, since the latter involve somewhat different MEA sizes.

[33] Recent theoretical development for the principle has focused mainly on the production and demand externalities. In the context of the so-called New Economic Geography models, such externalities can be shown to lead to the formation of industrial agglomerations consistent with the Hierarchy Principle (Fujita, Krugman and Mori [20], Tabuchi and Thisse [50]).

If this principle holds exactly, then it should be clear that for any given number of industry-choice MEAs, the average size will always be as large as possible. Hence the closeness of average employment sizes to the "upper bound" in Figure 3.5 already suggests a strong confirmation of this principle. Moreover, if average employment size is replaced by average population size, then essentially the same picture emerges (see Fig. 5.1). But such observations do not constitute a formal statistical test of this principle. Hence, as in the case of the NAS Rule above, it is desirable to construct an appropriate null hypothesis of "purely random industrial hierarchies" and to test this hypothesis against the observed data.

In doing so, however, one encounters several problems. First of all, the Hierarchy Principle is defined with respect to a given population distribution among MEAs. But any attempt to relocate population along with industries will necessarily change that distribution, and hence create certain ambiguities in the interpretation of the Hierarchy Principle itself. Moreover, as was seen in the test of the NAS Rule above, there is no obvious way to relate the relocation of population to that of industry. Hence it appears that there is a need for an alternative approach to testing this principle. The second problem encountered is that unlike the NAS Rule above, where the values of average employment sizes of industry-choice MEAs provide natural test statistics, the presence or absence of hierarchies is not measurable by any single numerical value. Hence the construction of statistical tests is somewhat more problematic. We now address these two problems in turn.

4.1 Hierarchies in terms of industrial diversity

As in the case of the NAS Rule above, one could replace populations with employment clusters, and redefine industrial hierarchies in terms of total employment size. However, a major shortcoming of this approach in our view is that it ignores regional structure. For example "Tokyo" and "Osaka" are treated simply as abstract locations when reallocating industries. Hence we believe that it is more appropriate to construct tests which preserve regional structure and attempt to identify the presence or absence of industrial hierarchies within that structure. With this in mind, we choose here to focus on the given structure of *industrial diversity* among regions rather than population per se.[34] This approach has several advantages. First of all, as will be seen below, this leads to a very natural random sampling procedure for testing the Hierarchy Principle. In particular, if m industries are present in a given MEA then one may regard this MEA as having m "slots" to which industries can be randomly assigned. This ensures, for example, that Tokyo and Osaka will continue to be major industrial centers in each random sample.

The second advantage of this approach is that industrial diversity of MEAs is in reality very closely related to their population size. Figure 4.1 shows this relationship for MEAs in 1999/2000.[35] The

[34] Refer to Section 2.2 for our definition of industrial diversity of a given region.
[35] A similar plot can be obtained for 1980/1981.

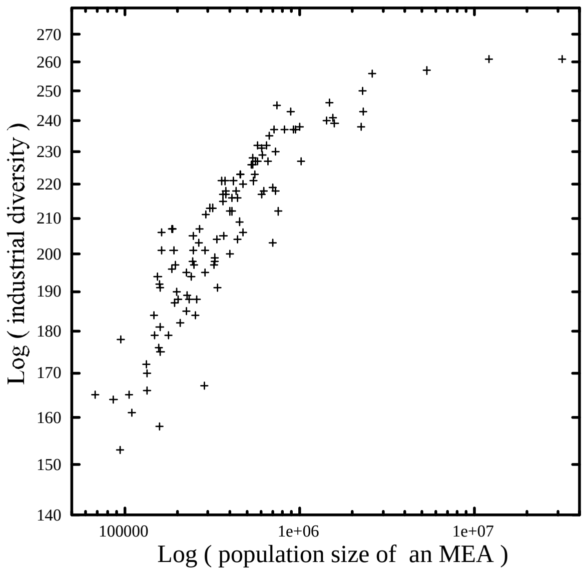

Figure 4.1: Population size and industrial diversity of MEAs

corresponding log-linear regression for the 1999/2000 data is shown below.[36]

$$\log(\mathrm{DIV}) = \underset{(0.03671)}{1.574}^{**} + \underset{(0.006591)}{0.1331}^{**} \log(\mathrm{SIZE}), \quad R^2 = 0.7907. \tag{4.1}$$

While the R^2 indicates that some unexplained dispersion remains, nevertheless the significance of the slope coefficient rivals that of the NAS Rule above (with P-value virtually zero).[37] In addition, the Spearman's rank correlation between the size and diversity is greater than 0.9 for both 1980/1981 and 1999/2000. Thus, there is close agreement between the diversity- and the size-rankings of MEAs.[38]

For these reasons, we now focus on industrial diversity, and hence redefine the Hierarchy Principle for testing purposes as follows. An industrial location pattern is said to satisfy the *Hierarchy Principle* iff industries found in a given MEA are also found in all MEAs with diversities at least as large.[39] Following Christaller [5], we call an industry which locates in a smaller [resp. larger] number of MEAs a *higher-order* [resp., *lower-order*] industry, and call a more [resp., less] diversified MEA a *higher-order* [resp.,

[36] There is no significant difference in the estimated coefficients between 1980/1981 and 1999/2000.

[37] The most diversified three MEAs (Tokyo, Osaka and Nagoya) are excluded from this regression. These are almost fully diversified within the present industrial classification. In other words, their industrial compositions cannot be distinguished within this industrial classification, and hence, the relation between population size and industrial diversity for these MEAs appears as qualitatively different from that of less diversified MEAs (refer also to footnote 22).

[38] It is to be noted that this correlation between size and diversity of MEAs appears to be stronger than similar correlations obtained using alternative indices reported in the literature. For instance, the Spearman's rank correlation between size and diversity for our MEAs in 1999/2000 is only 0.49 when diversity is measured by HHI, and 0.72 when the RDI is used, where the definitions of HHI and RDI are given in Section 2.2. However, these low correlations may be in part accounted for by the particular properties of these indices discussed in Section 2.2.

[39] It should be noted that from a formal viewpoint, this principle might be designated as the *Weak Hierarchy Principle,* since it is strict weakening of the classical Hierarchy Principle. In particular, if industries in a given MEA always occupy MEAs populations at least as large, then these MEAs will always have at least as many industries, i.e., at least as much diversity. Thus the classical Hierarchy Principle implies this Weak Hierarchy Principle. However the converse is false. For example if it were true that industries in a given MEA always occupy MEAs with populations at least as *small*, then while the classical Hierarchy Principle obviously fails, the Weak Hierarchy Principle continues to hold (since MEAs with smaller population will always have at least as much diversity as those with larger population). However, for sake of simplicity, we choose to ignore this formal distinction and refer to this weaker version as the "Hierarchy Principle".

lower-order] *MEA*.[40]

In these terms, the presence of an industrial hierarchy can be observed for MEAs in Japan in 1999 as in Figure 4.2 below.[41] Here MEAs are ordered on the horizontal axis by their industrial diversities (i.e., numbers of industries they contain), and industries are ordered on the vertical axis by their numbers of industry-choice MEAs. Hence each point "+" in the figure represents the event that the MEA in that column contains the industry in that row. Notice that the points are more sparse near the southwest

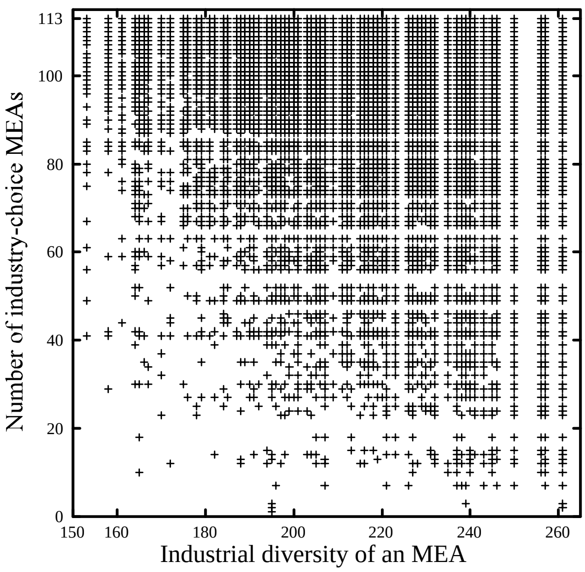

Figure 4.2: Hierarchy Principle (1999/2000)

corner, meaning that the industries with a smaller number of locations are found mainly in MEAs with large industrial diversity. On the other hand, MEAs with smaller industrial diversity tend to have more ubiquitous industries (i.e., those locating in a large number of MEAs). While this observation is already highly suggestive of the presence of the Hierarchy Principle, they in no way constitute a *test* of the Hierarchy Principle. Moreover, it should be clear that by pure chance alone, any given industry is more likely to be in those MEAs with higher diversity, i.e., with greater numbers of industries. Hence it is important to discriminate statistically between these random hierarchical effects and genuine hierarchies. Section 4.2 below develops a formal test of the Hierarchy Principle.[42]

[40] Higher-order industries are typically either subject to large scale economies (e.g., coke, briquette, blast furnace manufacturing, petroleum refining) or highly specialized (e.g., fur, leather, surveying equipment, fireproof product, spectacle manufacturing, special school education services, social and cultural science research services). Lower-order industries are subject to high tranport cost in the general sense. Examples of these are the manufacture and wholesale/retail of perishable products (e.g., meat and dairy food, vegetable and fruit food), that of heavy product (e.g., stone and related product, cement), and services/retails of frequent use (e.g., attorney services, department strores, automobile maintenance, drug and cosmetic retail). The detailed result is available from the authors upon request.

[41] Data for 1981 yields a similar plot.

[42] Though the Hierarchy Principle makes no assertion about the relative employment sizes of industries in each MEA, it is natural to expect that under this principle the employment within any given industry should be greater in larger MEAs. To check this, we calculated Spearman's rank correlations between the employment size of each industry in an MEA and the population size as well as diversity of an MEA. This correlation only fails to be significant (at the 5% level) for population size and/or diversity in 33 [resp., 18], of the 264 industries in 1980/1981 [resp., 1999/2000]. The average rank correlations are 0.61 and 0.50 [resp., 0.62 and 0.61] with population size and diversity, respectively, in 1980/1981 [resp., 1999/2000].

4.2 A test of the Hierarchy Principle

As stated above, our approach here is to construct an appropriate test statistic for measuring "degree of hierarchy," and to formulate an appropriate null hypothesis of "purely random hierarchies" in terms of this statistic.

4.2.1 The hierarchy-share statistic

We begin with the construction of a test statistic. To do so, note first that each cell containing a + in Figure 4.2 above can be viewed as part of a larger "hierarchy event" if all cells to the right also contain a +, i.e., if that industry is contained in all MEAs with diversities at least as large. Hence if we assign a value 1 to each cell with a + whenever it is part of a hierarchy event, and assign a value 0 otherwise, then we can use these binary outcomes to construct an appropriate test statistic. In particular, the total of these values must lie between zero and the total number of +'s. Hence dividing by this maximum total, we obtain a natural test statistic, P, representing the fraction of hierarchy events that occur. This *hierarchy share,* P, must always lie between 0 and 1, and must achieve its maximum value, $P = 1$, exactly when the Hierarchy Principle holds. Thus P constitutes a natural test statistic for measuring "closeness" to the Hierarchy Principle.

To formalize these ideas we begin with a set of industries, $i \in \mathbf{I} = \{1, \ldots, I\}$, and a set of MEAs, $r \in \mathbf{R} = \{1, .., R\}$. The industry-choice MEAs for industry i then correspond to some subset, $\mathbf{R}_i \subseteq \mathbf{R}$, and the number of these MEAs is given by the cardinality, $n_i = \#\mathbf{R}_i$. We assume (as in Figure 4.2) that industries are ranked by their choice numbers so that for any $i, j \in \mathbf{I}$, $i > j \Rightarrow n_i \geq n_j$. Similarly, if the number (diversity) of industries in a given MEA, r, is denoted by D_r, then we assume that MEAs are ranked by these values so that for all MEAs, $r, s \in \mathbf{R}$, $r > s \Longrightarrow D_r \leq D_s$. Hence MEAs are assumed to be ordered from the largest diversity ($r = 1$) to the smallest diversity ($r = R$), and the ordered set, $D = (D_r : r \in \mathbf{R})$, is designated as the corresponding *diversity structure.* In this context, each cell of Figure 4.2 corresponds to a pair (n_i, D_r), which we now represent simply by (i, r).

Thus, the above claim does hold approximately, though it is by no means universal.

One possible reason for the imperfect correlation between size/diversity and industry employment in MEAs is that the Hierarchy Principle itself does not hold perfectly in reality (as we will see below). Some medium size MEAs are highly specialized in a few industries that deviate from the Hierarchy Principle, and correspond more closely to Henderson's "specialized cities"[23] [24]. Henderson emphasizes industry-specific localization economies as a source of city formation. In addition, he notes [25] that medium/small size cities (with population size smaller than 500,000) tend to exhibit the strongest specialization. In 1999/2000, more than 10% of total employment was concentrated in a single MEA for 48 industries, among which 17 are in small/medium size MEAs. Typically, the specialization of these MEAs is based on immobile resources (e.g., wooden material and liquor manufacturing), historical background (e.g., explosive assembling), or plant-level scale economies in production (e.g., steel manufacturing, metal and alloy rolling).

Finally, it is of interest to note that the variations in the rank correlations above themselves exhibit a pattern, namely that more ubiquitous industries show high correlations, while more localized ones show only low correlations. This relation may in part be due to trade patterns between MEAs. In particular, it is natural to expect that industries in smaller (i.e., less diverse) MEAs will be more export-oriented than those in larger (more diverse) MEAs. If so, then the employment in these export-oriented industries should tend to be large relative to the population sizes of their MEAs.

Next we define a *location indicator*, x_{ir}, for each industry, $i \in \mathbf{I}$, and MEA, $r \in \mathbf{R}$, by

$$x_{ir} = \begin{cases} 1 & , \text{ if industry } i \text{ locates in } r \\ 0 & , \text{ otherwise} \end{cases} . \tag{4.2}$$

For the given *observed diversity structure*, $D_0 = \{D_{0r} : r \in \mathbf{R}\}$, each vector, $x = (x_{ir} : i \in \mathbf{I},\ r \in \mathbf{R})$, of indicator variables satisfying $\sum_{i\in\mathbf{I}} x_{ir} = D_{0r}$ for all $r \in \mathbf{R}$, is designated as a *feasible location pattern* for D_0. In particular, the *observed location pattern* given by the empirical data is obviously feasible, and is denoted by $x^0 = (x^0_{ir} : i \in \mathbf{I},\ r \in \mathbf{R})$.

For each feasible location pattern, x, and ir-pair, we let $S_{ir} = \{s \in \mathbf{R} : D_s \geq D_r\}$, then one can define the *hierarchy event,* $H_{ir}(x)$, by

$$H_{ir}(x) = \begin{cases} 1 & , \text{ if } x_{is} = 1 \text{ for all } s \in S_{ir} \\ 0 & , \text{ otherwise} \end{cases} , \tag{4.3}$$

so that the event *occurs* when $H_{ir}(x) = 1$. Observe that the event $H_{ir}(x)$ is only *feasible* (i.e., can only occur) for ir-pairs with $x_{ir} = 1$. If the number of *feasible hierarchy events* is denoted by $h = \sum_{r\in\mathbf{R}} D_{0r} = \sum_{ir} x_{ir}$, then the fraction of hierarchy events in x which occur (i.e., are consistent with the Hierarchy Principle) is given by

$$P(x) = \frac{1}{h} \sum_{ir} H_{ir}(x). \tag{4.4}$$

and is designated as the *hierarchy share* for x. In particular, the value, $p_0 = P(x^0)$, denotes the *observed hierarchy share* derived from the given data.

4.2.2 The testing procedure

In terms of this statistic, we next formulate an appropriate procedure for testing the Hierarchy Principle as follows. In the absence of any additional hierarchical structure, our fundamental hypothesis is that all location patterns consistent with the observed diversity structure, D_0, should be *equally likely.* Hence if the set of feasible location patterns for D_0 is denoted by

$$\mathbf{X}_0 = \left\{ x = (x_{ir} : i \in \mathbf{I},\ r \in \mathbf{R}) : \sum_{i\in\mathbf{I}} x_{ir} = D_{0r} \ ,\ r \in \mathbf{R} \right\} \tag{4.5}$$

and if the elements of $\mathbf{X}_0$ are regarded as possible realizations of a random vector, $X = (X_{ir} : i \in \mathbf{I},\ r \in R)$, then our null hypothesis is simply that

$$H_0 : X \ \textit{is uniformly distributed on} \ \mathbf{X}_0 \tag{4.6}$$

In theory, one would then like to derive the distribution of the hierarchy-share statistic, $P(X)$, and state this hypothesis directly in terms $P(X)$. The appropriate test of the Hierarchy Principle would then amount to determining how likely it is that the observed hierarchy share, p_0, came from this statistical population. However, in the present case it is far simpler to simulate many random draws from X_0 under

H_0, and then construct the corresponding *sampling distribution* of $P(X)$. In particular, to draw a random sample x from X_0 under H_0, one may proceed as follows:

(i) For each MEA, $r \in \mathbf{R}$, randomly sample a subset of D_{0r} industries from $\mathbf{I}$ without replacement [say by randomly permuting the set of labels $\mathbf{I} = \{1, .., I\}$, and taking the sample to be the first D_{0r} elements from this list].

(ii) Set $x_{ir} = 1$ if industry i is in the sample drawn for r, and $x_{ir} = 0$ otherwise.

It should be clear that this procedure will automatically yield a feasible location pattern $x \in \mathbf{X}_0$, and that all such patterns will be equally likely. So to construct a set of M independent samples of the hierarchy-share statistic, say $\{P_m : m = 1, .., M\}$, under H_0 :

(iii) Construct sample location patterns, $\{x^m : m = 1, .., M\}$ using (i) and (ii), and set $P_m = P(x^m)$, $m = 1, .., M$.

Finally to carry a test of the Hierarchy Principle, observe that one may estimate the cumulative distribution function, $F(p) = \Pr(P < p)$, under H_0 by

$$\hat{F}(p) = \frac{1}{M} \#\{m : P_m < p\} \tag{4.7}$$

where $\#\{m : P_m < p\}$ represents the number of samples with $P_m < p$. Following standard testing procedures, one can then estimate the appropriate *P-value* for the test by

$$\widehat{\Pr}(P \geq p_0) = 1 - \hat{F}(p_0) \tag{4.8}$$

This estimates the probability of getting a hierarchy share at least as large as the observed fraction, p_0, under H_0. If M is sufficiently large to ensure a good approximation in (4.8), and if this P-value is sufficiently small, say $< .01$, then one can safely conclude that the actual data is much closer to the Hierarchy Principle than would be expected under H_0.

In summary, this testing procedure appears to offer three important advantages: (i) the test is based on a simple summary statistic that reflects the "degree of closeness" of location patterns to the Hierarchy Principle, (ii) it requires only simple random sampling for the required simulations, and (iii) it is completely nonparametric and makes no appeal to asymptotic distribution theory.

4.3 Test results

Within this testing framework, we calculated the hierarchy shares for 1000 randomly generated location patterns under H_0. The sampling distribution (histogram) of P obtained from this sample based on the diversity structure in 1999 is shown in Figure 4.3 (with sample mean 0.15 and standard deviation 0.004). In addition, the observed hierarchy share for the empirical location pattern in 1999 ($p_0 = 0.71$) is indicated by the vertical broken line in the figure. As it is obvious from the figure, this hierarchy share is vastly higher than that those of the random samples. In fact, there is no hierarchy share for a random sample which comes even close to that of the empirical location patterns, and the estimated P-value is virtually zero in each case.

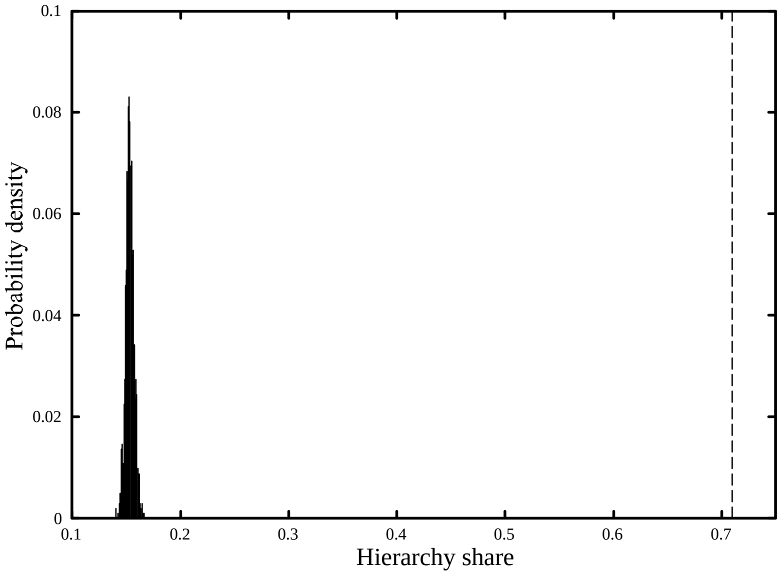

Figure 4.3: Sampling distribution of hierarchy shares under H_0

4.4 Alternative testing procedures

As mentioned above, there are many possible approaches to testing this Hierarchy Principle. Here we mention two other possibilities that we have explored. First observe that our emphasis in the above approach has been to preserve regional structure in terms of industrial diversity. Here industries themselves were treated as completely interchangeable under the null hypothesis, and were randomly assigned to MEAs in any manner consistent with the given levels of industrial diversity. An alternative approach is to focus on industries, and to preserve the localization structure of industries rather than the diversity structure of MEAs. In this case one would start with an *observed localization structure*, $n^0 = \{n_i^0 : i \in \mathbf{I}\}$, of industry-choice MEA numbers for each industry, and [as a parallel to (4.5) above] would define the set of *feasible location patterns*, $\mathbf{X}^0$, consistent with n^0 by

$$\mathbf{X}^0 = \left\{ x = (x_{ir} : i \in \mathbf{I},\ r \in \mathbf{R}) : \sum_{r \in \mathbf{R}} x_{ir} = n_i^0\ ,\ i \in \mathbf{I} \right\} \tag{4.9}$$

In this context, the appropriate null hypothesis, H^0, now take the form:

$$H^0 : X \textit{ is uniformly distributed on } \mathbf{X}^0 \tag{4.10}$$

One can then construct random samples from $\mathbf{X}^0$ in a manner paralleling the procedure above. Here for each industry, $i \in \mathbf{I}$, one randomly samples a subset of n_i MEAs and defines $x_{ir} = 1$ iff r is in the assignment for industry i. The construction of hierarchy shares, $P(X)$, is the same as before. Hence the appropriate test of H^0 amounts to estimating the P-value of the observed hierarchy share, p_0, above under the new sampling distribution obtained from these randomly generated hierarchy shares.

We have carried out such a test, and the results are shown in Figure 4.4 (with sample mean 0.56 and standard deviation 0.007). Here it is clear that observed hierarchy share, p_0, in 1999 continues to be vastly greater than any of the randomly sampled shares, and hence that this test also provides strong evidence in favor of the Hierarchy Principle versus H^0.[43] However, one interesting difference between

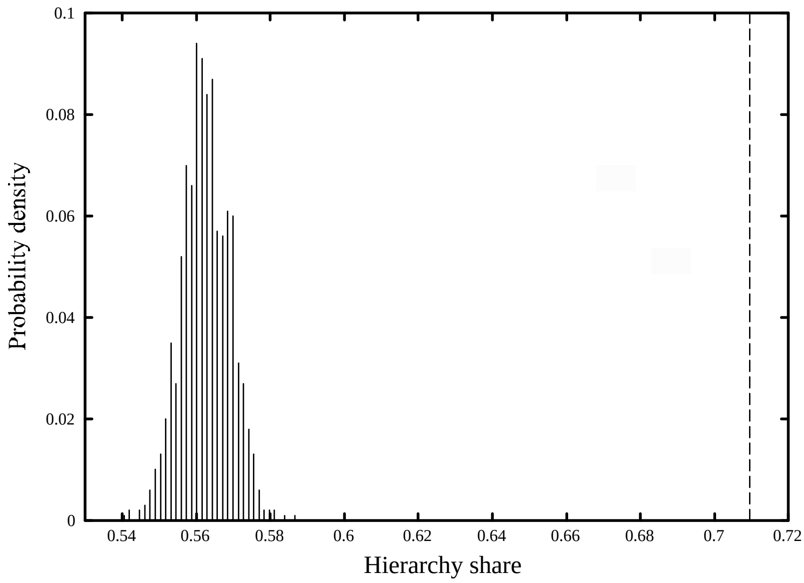

Figure 4.4: Sampling distribution of hierarchy shares under H^0

these two test results is that the sampling distribution of hierarchy shares is much larger under H^0 than under H_0. This at first seems surprising since H^0 ignores all regional structure, and treats MEAs as completely interchangeable. However, further reflection shows that the key difference here relates to ubiquitous industries. In Japan, approximately 52% of the industries are located in more than 90% of all MEAs. Hence when the observed location structure, n^0, is held fixed, there must necessarily be a large number of hierarchy events associated with these ubiquitous industries in every random sample. It can thus be inferred that when location structure is randomized (as under H_0), this leads to a much smaller range of n_i-values than is actually observed. [By looking at the average percent occupancies for industries over the 1000 samples shown in Figure 4.3, it turns out that the *most ubiquitous* industry occupies less than 90% of all MEAs, and the *most localized* industry occupies more than 60% of all MEAs.] Conversely, if the observed diversity structure, D_0, is randomized (as under H^0), our results show that this leads to a much smaller range of D_r-values than is actually observed. Hence, hierarchy events are more likely under H^0 than under H_0. But, notice also in any case that *both MEAs and industries are in reality much more*

[43] Again, essentially the same result is obtained for the case in 1981.

heterogeneous than would be expected in these "randomized" worlds.

A second approach to testing is motivated by the fact that the hierarchy shares defined in (4.3) are in fact rather stringent. In particular, consider two industries, $i, j \in \mathbf{I}$, that are each located in all but one MEA. If the MEA not containing i were one of the smallest, then this would generate many hierarchy events for industry i. But if the MEA not containing j happened to be one of the largest, then there would be very few hierarchy events for j. Hence even though both industries are located in almost the same set of MEAs, one is highly consistent with the Hierarchy Principle while the other is not. This extreme example suggests that the 0-1 hierarchy-event measure, H, should perhaps be replaced with a more graduated measure of "closeness" to pure hierarchy events. One obvious measure of this type looks at the *fraction* of possible location events that actually occur "at or above" any given event. More precisely, if for any location event, x_{ir}, we let $s_{ir} = \#S_{ir}$, then the desired *fractional hierarchy event*, $\overline{H}_{ir}(x)$, is defined for each location pattern x by

$$\overline{H}_{ir}(x) = \frac{1}{s_{ir}} \sum\nolimits_{k \in S_{ir}} x_{ik}$$

One can then average these values to produce an overall test statistic, $\overline{P}(x)$, of *fractional hierarchy shares.* A test of H_0 using $\overline{P}$ rather than P produces results that are qualitatively the same as those above. As expected, the values of $\overline{P}$ are uniformly higher than those of P, but the observed values, $\overline{p}_0$, are always well above all sampled values. Hence this test provides even further support for the Hierarchy Principle.

Hence, we conclude that the actual location pattern of the three-digit industries in Japan is highly consistent with the Christaller's Hierarchy Principle, i.e., the actual industrial hierarchy is highly non-random even after controlling for the relative industrial diversities among MEAs, as well as for the relative number of industry-choice MEAs among industries.

Finally, it should be noted that the notion of "hierarchy" itself depends on the degree of industrial aggregation being considered. To see this, consider the most disaggregated case in which each establishment constitutes a single industry category. Then obviously the principle can never hold, since no establishment can be located at more than one place. At the opposite extreme, if all industries are aggregated into a single industry category (so that metro areas can have only this industry), then the principle must hold trivially. In the case of Japan (with establishment location data publicly available up to three-digit level) there is seen to be a wide variation in the number of metro areas in which a given industry is present. Thus Japan provides an interesting case for testing this principle.[44]

[44] More generally, it would be of interest to identify the level of industrial aggregation at which the Hierarchy Principle is most pronounced. Since this principle directly relates the population size of a metro area to its industrial composition, knowledge of the "best" industry classification would appear to be useful for the formation of appropriate industrial policies at the metro area level. However, such analyses require the use of spatially disaggregated industrial data, and are thus beyond the scope of this paper.

5 Industrial location and the Rank-Size Rule

Given the strong evidence above for both the NAS Rule and the Hierarchy Principle, we now consider a possible relationship between these two empirical regularities. As stated in the introduction, it turns out that in the presence of the Hierarchy Principle, the NAS Rule is essentially identical to perhaps the most widely known empirical regularity in all of economic geography, namely the *Rank-Size Rule* (or Zipf [51]'s Law) for city size distributions. This rule asserts that if cities *in a given country* are ranked by (population) size, then the rank and size of cities are log-linearly related:

$$\log SIZE = \sigma - \theta \log RANK. \tag{5.1}$$

Moreover, the estimate of θ is usually close to one. The most complete cross-country comparison for this relationship was conducted by Rosen and Resnick [45] who reported that in 1970 the estimates of θ for 44 countries lie between 0.51 and 1.24 with an average value 0.90 and standard deviation, 0.14.[45] It is to be noted, however, that the definition of "city" in their analysis is the administrative city, which often fails to represent cities in economic sense. They show for countries where data for metro area exists, the estimates of θ are even closer to 1 when metro areas are used instead of administrative cities.[46] For our Japanese MEAs, we have obtained the estimated values, $\theta = 0.95$ ($R^2 = 0.95$) for 1980 and $\theta = 1.00$ ($R^2 = 0.95$) for 2000. Although there is no definitive evidence for the Rank-Size Rule, the recent literature suggests that observed rank-size relationships are too close to this rule to be dismissed as completely irrelevant.[47]

To establish a relationship between this rule and the NAS Rule in the presence of the Hierarchy Principle, it is useful to begin with a simple formulation using a continuum of population units (MEAs). Here the essence of the connection is mathematically transparent, though the notions of "population size" and "rank" are somewhat tenuous. We then develop a more realistic (but mathematically more complex) version of this relationship in terms of discrete population units.

5.1 A continuous model

Consider a continuum of *population units* (MEAs) on the interval $\mathbf{R} = [0, R]$, where each population unit, $r \in \mathbf{R}$, is ranked by its *size*, $\rho(r)$, with $r < r' \Leftrightarrow \rho(r) > \rho(r')$. The largest population unit is represented by rank 0, the smallest by rank R, and the total number (mass) of population units by $R = \int_{\mathbf{R}} dr$. Consider also a set of *industry types*, $i \in \mathbf{I}$, with each industry i occupying a (measurable) subset, $\mathbf{R}_i$, of

[45] In Rosen and Resnick [45], $\log RANK$ is regressed against $\log SIZE$.

[46] As Eeckhout [16] pointed out, the "log-lineality" which appears in both the Rank-Size Rule and NAS Rule could be due to the fact that we only look at the upper tail of the city-size distribution. If all the tiny towns and villages are taken into account, this log-linearity may not continue to hold. Nevertheless, the stable regularity that appears among the largest cities is still remarkable. Moreover, if scale economies play a major role in the formation of a metro area, then it is not surprising to find fewer tiny cities than predicted by the Rank-Size Rule. Indeed, there may be a "critical population size" required for a viable city structure. If so, then it might be most appropriate to study population concentrations above and below this critical size as separate types of concentrations.

[47] See, e.g., Black and Henderson [3], Dobkins and Ioannides [8], Ioannides and Overman [27], and a recent more extensive discussion by Gabaix and Ioannides [22].

the population units in $\mathbf{R}$, and let the number (mass) of those units be denoted by $n_i = \int_{\mathbf{R}_i} dr$. Assuming that ρ is continuous on $\mathbf{R}$, it follows that *average size*, $\overline{R}_i$, of those units occupied by i is given by

$$\overline{R}_i = \frac{1}{n_i}\int_{\mathbf{R}_i} \rho(x)dx. \tag{5.2}$$

In such an economy, $\mathbf{E} = (\mathbf{R}, \mathbf{I}, \rho, n)$, the log-linear relationships in expressions (3.1) and (3.2) now take the form

$$\ln(\overline{R}_i) = a - \beta \ln(n_i), \qquad a, \beta > 0 \tag{5.3}$$

where for convenience we ignore any random factors and use only simple equalities. This log-linear relation, which can also be written as

$$\overline{R}_i = \alpha n_i^{-\beta}, \qquad \alpha = \exp(a) > 0, \beta > 0 \tag{5.4}$$

is now designated (in a manner similar to Section 3.1.1) as the *Number-Average Size* (*NAS*) *Rule* with scale factor α and exponent β for industrial locations in economy $\mathbf{E}$.

The *Hierarchy Principle* for economy $\mathbf{E}$ then asserts that for each industry type, $i \in \mathbf{I}$, the population units occupied by i are an *interval* in $\mathbf{R}$ of the form $\mathbf{R}_i = [0, r_i]$, where r_i denotes the rank of the smallest population units occupied by i. Hence the total number, n_i, of units occupied by i is now given by r_i, so that the average size in (5.2) is given by:

$$\overline{R}_i = \frac{1}{r_i}\int_0^{r_i} \rho(r)dr. \tag{5.5}$$

Finally, we recall that population units, $r \in \mathbf{R}$, are said to satisfy a *Rank-Size Rule* with scale factor s and exponent θ if and only if :

$$\rho(r) = sr^{-\theta}, \qquad s, \theta > 0 \tag{5.6}$$

[where $s = \exp(\sigma)$ in expression (5.1) above]. With these definitions, our main result is to show that in the presence of the Hierarchy Principle, the NAS Rule for industrial locations is essentially equivalent to the Rank-Size Rule for population units, and in addition that the exponents β and θ must be the same and less than one. In particular, if we now say that $\mathbf{I}$ is a *full set of industries* for economy $\mathbf{E}$ whenever there exists for each population unit, $r \in \mathbf{R}$, at least one industry, $i \in \mathbf{I}$, with $r_i = r$, then:

Theorem 1 *For any economy $\mathbf{E} = (\mathbf{R}, \mathbf{I}, \rho, n)$ satisfying the Hierarchy Principle and any $\beta \in (0,1)$,*
(i) if population units in $\mathbf{E}$ satisfy the Rank-Size Rule with scale factor α and exponent β, then industrial locations satisfy the NAS Rule with scale factor $\alpha/(1-\beta)$ and exponent β.
(ii) Conversely, if $\mathbf{I}$ is a full set of industries for $\mathbf{E}$, and if industrial locations satisfy the NAS Rule with scale factor α and exponent β, then population units satisfy the Rank-Size Rule with scale factor $\alpha(1-\beta)$ and exponent β.

Proof: (i) First suppose that population units satisfy the Rank-Size Rule with scale factor α and exponent $\beta \in (0,1)$. Then by the Hierarchy Principle together with (5.5) it follows that for each industry $i \in \mathbf{I}$,

$$\begin{aligned} \overline{R}_i &= \frac{1}{n_i}\int_0^{r_i} \rho(x)dx = \frac{1}{n_i}\int_0^{r_i} \alpha x^{-\beta}dx \\ &= \frac{\alpha}{n_i}\left[\frac{1}{1-\beta}x^{1-\beta}\right]_0^{r_i} = \frac{\alpha}{1-\beta}n_i^{-\beta} \end{aligned} \tag{5.7}$$

and hence that industrial locations satisfy the NAS Rule with scale factor $\alpha/(1-\beta)$ and exponent β.

(ii) Next suppose that $\mathbf{I}$ is a full set of industries for $\mathbf{E}$ and that industrial locations satisfy the NAS Rule with scale factor α and exponent $\beta \in (0,1)$. Then for each $r \in \mathbf{R}$ there is some industry $i \in \mathbf{I}$ with $r = r_i = n_i$, so that by the Hierarchy Principle together with (5.4) and (5.5) we obtain the following identity for all $r \in \mathbf{R}$

$$\frac{1}{r}\int_0^r \rho(x)dx = \overline{R} = \alpha r^{-\beta} \;\; \Rightarrow \;\; \int_0^r \rho(x)dx = \alpha r^{1-\beta} \tag{5.8}$$

Finally, by differentiating this identity in r it follows that

$$\rho(r) = \alpha(1-\beta)r^{-\beta} \tag{5.9}$$

and hence that population units satisfy the Rank-Size Rule with scale factor $\alpha(1-\beta)$ and exponent β. ■

The key feature of this proof is its obvious simplicity. In addition, it allows both the Rank-Size Rule and NAS Rule to be expressed as exact power functions. Finally it shows that this equivalence is only meaningful for exponents in the open unit interval, $\beta \in (0,1)$.[48] In particular it reveals the fundamental differences between $\beta < 1$, $\beta = 1$, and $\beta > 1$. For the "classic" Rank-Size Rule, $\beta = 1$, the integral in (5.7) becomes infinite and right hand side of (5.8) reduces to a constant. The behavior of $\beta > 1$ is seen from (5.7) and (5.9) to be even worse in terms of its implied relation between the Rank-Size Rule and NAS Rule.

But while this proof is quite enlightening, the continuous model itself is plagued by conceptual difficulties. There is of course the well-known problem of interpreting a continuum of population units (i.e., interpreting ρ as a density). But even more serious is the fact that the choice of rank 0 for the largest population unit implies from (5.6) that the population $\rho(0)$ must be *infinite*. One could of course modify the above model by choosing a positive rank value, r_0, for the highest order population unit. However, an analysis of this case shows that the simplicity of the above relations all but disappear.[49] So we choose to leave this continuous model as it is.

A more realistic discrete approach is developed below which overcomes many of these problems, but requires a more delicate analysis. In addition this discrete approach requires weaker formulations of

[48] As mentioned above, the estimate of θ in (5.1) is not significantly different from one for the case in 2000. Refer to Section 5.3 for possible explanations.

[49] Note however that for both the classic case, $\beta = 1$, and the case $\beta > 1$, a positive value of r_0 does have the advantage of removing the infinite-integral problem. But neither of these cases yield a log-linear expression for $\overline{R}$ in terms of n. Moreover, for $\beta > 1$ the resulting expression is not even monotone decreasing in n. Hence the desired relationship still fails to hold in these cases.

the basic Rank-Size Rule and NAS Rule which focus on the right tail of "sufficiently small" population units. Hence it is our belief that the continuous formulation above reveals in the simplest possible way the essential relationship between the NAS Rule, Hierarchy Principle, and Rank-Size Rule. Nonetheless, it turned out that the results based on this discrete model provides more direct basis for the empirical analysis in Section 5.3 below. For this reason, it is worth establishing this relationship under the discrete setup.

5.2 A discrete model

In the following development we shall alter the above terminology in a manner more suitable for analysis. For purposes of this section we now take the set of population units (MEAs) to be the set of *positive integers*, $\mathbf{R} = \{1, 2, ..\}$, and replace the above continuum of population units with a *ranked population sequence*, $N = (N_r : r \in \mathbf{R})$, consisting of positive population values subject to the ranking: $N_r \geq N_s$ whenever $r < s$. Hence the largest population has rank 1 and so on. There are of course only finitely many population units in any real world setting, so that actual populations can be interpreted as *initial segments* $(N_1, .., N_n)$ of ranked population sequences. For every population sequence there is a uniquely associated *upper-average sequence*, $\widetilde{N} = (\widetilde{N}_r : r \in \mathbf{R})$, defined for all $r \in \mathbf{R}$ by

$$\widetilde{N}_r = \frac{1}{r} \sum\nolimits_{s=1}^{r} N_s \tag{5.10}$$

Since averages of a monotonically nonincreasing sequence are also monotonically nonincreasing, it follows that $\widetilde{N}$ is also a ranked population sequence. More importantly, note that expression (5.10) is precisely the discrete analogue of expression (5.5) above, and hence that role of the Hierarchy Principle in the present context is to focus interest on the connections between these two ranked population sequences.

The corresponding discrete analogues of the Rank-Size and NAS Rules assert that the r^{th} terms of N and $\widetilde{N}$, respectively, are negative power functions of r. However, should be clear from (5.10) that N_r and $\widetilde{N}_r$ *cannot* simultaneously be power functions of r (since sums of power functions are not power functions). But the continuous case above suggests that some connection of this type ought to be possible. The key idea here is to replace power functions with the weaker notion of "asymptotic power laws":

Definition 2 *For any ranked population sequence, $N = (N_r : r \in \mathbf{R})$, and exponent, $\beta > 0$,*

*(i) N is said to satisfy an **asymptotic β-power law** with scale factor $\alpha > 0$, iff the sequence of ratios*

$$\theta_r = \frac{N_r}{\alpha r^{-\beta}} \;\; , \; r \in \mathbf{R} \tag{5.11}$$

converge to unity, i.e., iff

$$\lim_{r \to \infty} \theta_r = 1 \tag{5.12}$$

(ii) If in addition it is true that

$$\lim_{r \to \infty} r \left[\theta_r - \left(\frac{r}{r-1} \right)^{\beta} \theta_{r-1} \right] = -\beta \tag{5.13}$$

then N is said to satisfy a ***strong asymptotic*** $\boldsymbol{\beta}$***-power law*** *with scale factor* α

Hence N satisfies an asymptotic β-power law iff its terms can be written as an approximate power function

$$N_r = \alpha r^{-\beta}\theta_r \tag{5.14}$$

with the approximation improving as r becomes large (i.e., as population units become small). The motivation for the stronger condition (5.13) is much less clear. This condition is analyzed in Appendix 7.3, where it is shown that condition (5.13) always holds for the limiting case in which all error factors θ_r are identically one. In this sense, (5.13) can be viewed as rate-of-convergence condition requiring that (θ_r) approach one "fast enough" to ensure that this limit relation continue to hold. In the Appendix it is shown that (5.13) will hold whenever this convergence is of order $o(1/r)$, [roughly, whenever (θ_r) converges to one faster than the sequence $1/r$ converges to zero]. This stronger convergence property highlights one key difference between asymptotic power laws for ranked population sequences N and their upper-average sequences $\widetilde{N}$. Since averaging is by nature a smoothing operation, it is reasonable to expect that any type of convergence for N should imply even faster convergence for $\widetilde{N}$. This intuition is confirmed by the following key result (proved in Appendix 7.3):

Theorem 3 *For any exponent, $\beta \in (0,1)$, a ranked population sequence N satisfies an asymptotic β-power law with scale factor α iff its upper-average sequence $\widetilde{N}$ satisfies a strong asymptotic β-power law with scale factor $\alpha/(1-\beta)$.*

To restate this result in a manner paralleling the continuous case above, it is convenient to make a number of simplifying assumptions. First observe that if two or more population units are of exactly the same size then population "rank" is somewhat ambiguous (particularly with respect to the Hierarchy Principle). So to avoid unnecessary complications we here model populations by a *strictly ranked population sequence*, $N = (N_r : r \in \mathbf{R})$, with population ranking: $r < s \Leftrightarrow N_r > N_s$. Next, we focus on economies with a *full* set of industries, $\mathbf{I}$, so that if $\mathbf{R}_i \subseteq \mathbf{R}$ denotes the set of population units occupied by industry i, then for each population unit r there is some $i \in \mathbf{I}$ with $r \in \mathbf{R}_i \subseteq \{1, 2, .., r\}$. For purposes of analysis, it is enough to focus on a single representative industry of this type, which we now denote as industry r. Hence associated with N is a unique *industrial-location sequence*, $\mathbf{L} = (\mathbf{R}_r : r \in \mathbf{R})$ with $r \in \mathbf{R}_r \subseteq \{1, 2, .., r\}$ for each $r \in \mathbf{R}$. For this representative economy, $\mathbf{E} = (N, \mathbf{L})$, we now say that population units satisfy a *Rank-Size Rule* with scale factor $\alpha > 0$ and exponent $\beta > 0$ iff the population sizes in N are of the form (5.14) with ratio sequence (θ_r) satisfying (5.12). Similarly, if for each set of industrial locations, $\mathbf{R}_r$, we let $n_r = \#\mathbf{R}_r$, so that the average population size of these locations is denoted by

$$\overline{N}_r = \frac{1}{n_r}\sum\nolimits_{s\in\mathbf{R}_r} N_s \tag{5.15}$$

then we now say that industrial locations in economy $\mathbf{E}$ satisfy a *Number-Average Size (NAS) Rule* with scale factor $\alpha > 0$ and exponent $\beta > 0$ iff for all industries $r \in \mathbf{R}$ the average population sizes $\overline{N}_r$ are of the form

$$\overline{N}_r = \alpha n_r^{-\beta} \theta_r \tag{5.16}$$

with ratio sequence (θ_r) satisfying (5.12). If in addition this ratio sequence satisfies the limit condition (5.13) then industrial locations in $\mathbf{E}$ are said to satisfy a *strong NAS law*. Finally, industries in economy $\mathbf{E}$ are said to satisfy the *Hierarchy Principle* iff the sets of industrial locations for all industries $r \in \mathbf{R}$ are given by the corresponding r^{th} initial segments of $\mathbf{R}$, i.e., iff

$$\mathbf{R}_r = \{1, 2, .., r\} \tag{5.17}$$

With these definitions, the desired equivalence result for the discrete case is given by the following corollary to Theorem 3:

Corollary 4 *For any economy* $\mathbf{E} = (N, \mathbf{L})$ *satisfying the Hierarchy Principle and any exponent* $\beta \in (0, 1)$, *population units in* $\mathbf{E}$ *satisfy a Rank-Size Rule with scale parameter* $\alpha > 0$ *and exponent* β *iff industrial locations in* $\mathbf{E}$ *satisfy a strong NAS Rule with scale parameter* $\alpha/(1-\beta)$ *and exponent* β.

Proof: We need only observe that in the presence of the Hierarchy Principle, (5.17), the sequence of average population sizes in (5.15) is precisely the upper-average sequence, $\widetilde{N}$, for the (strictly) ranked population sequence N. Hence the result is an immediate consequence of Theorem 3 with the Rank-Size Rule and NAS Rule treated as instances of asymptotic power laws. ■

5.3 Empirical evidence

For 1999/2000, Figure 5.1 shows plots of (a) NAS distribution: average size versus the number of industry-choice MEAs for each industry, (b) upper-average distribution: average size of the largest MEAs up to each rank, and (c) rank-size distribution: size versus rank of MEAs. Recall that plot (b) gives the upper bound for plot (a), and notice that for industries with the number of industry-choice MEAs greater than 10 (i.e., for a sufficiently large r relative to rank 1), the average size of the industry-choice MEAs is almost coincident with its upper bound (as in a manner similar to Figure 3.5 above). This re-confirm the fact that the industrial location is quite consistent with the Hierarchy Principle in this period.

Next, notice that plot (b) is almost log linear. Theorem 3 asserts that the log linearity of plot (b) could very well be a consequence of log linearity for the MEA rank-size distribution in plot (c), even though log linearity in (c) is not nearly as strong as in (b). One obvious source of the discrepancy is that while plot (b) and plot (c) must coincide at rank = 1, plot (c) should lie strictly below plot (b) for ranks ≥ 2. As stated in the theorem, the convergence to log linearity for plot (b) is expected to be *faster* than for plot (c). Thus, fitting a log-linear curve to rank-size distribution (c) would likely produce the estimated slope steeper than that for the upper-average plot (b). In fact, there is a clear difference between the slopes of the log-linear regression for plot (b) [~ 0.7] and plot (c)[~ 1.0]. In other words, if we are interested in the *limiting case* (i.e., for the case of a very large number of cities), the right curves to compare are (a) and (b), which indeed supports our theoretical result.

Another evident source of the failure of log linearity in plot (c) is the small MEAs with populations less than 300,000. That is, there are too few small cities than the Rank-Size Rule would suggest. The "dip" we see in the small MEAs of the rank-size distribution is in fact commonly observed in other countries (see, e.g., Black and Henderson [3], Rosen and Resnick [45], and Eeckhout [16]). It may be suggesting that fundamentally different mechanisms are at work in the formation of small cities. Especially if the scale economies play a major role for the formation of a city, it is reasonable to consider there is a critical size for a location to take off as a city. This leads to a rapid decrease in the number of cities below a certain size.[50]

But, for the rank-size range that appears reasonably log linear (i.e., if the largest three MEAs and those with smaller population than 300,000 are excluded), the estimate of the log-measured slope of the rank-size distribution (plot (c)) is -0.76 ($R^2 = 0.99$), which is seen to be much closer to the slope estimates for the NAS Rule in (3.1) and (3.2) above.

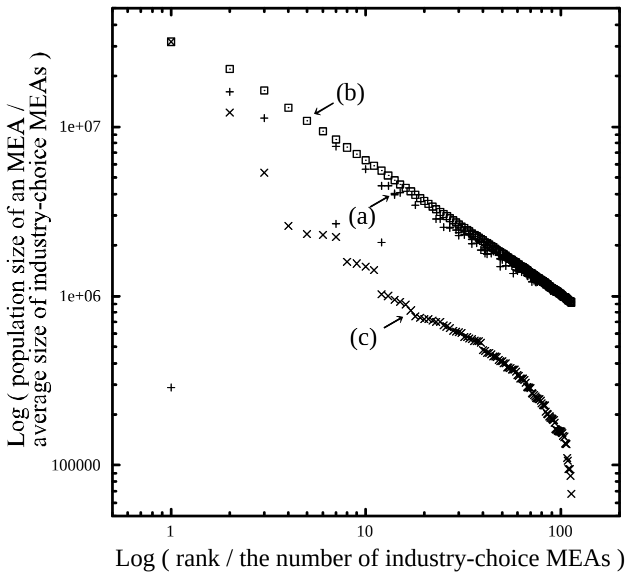

Figure 5.1: The Rank-Size Rule and the NAS Rule (1999/2000)

[50]Interestingly, a similar property is found in the size distribution of firms (see Negel, Shubik, Paczuski and Bak [39]).

6 Discussion and conclusions

In this paper, we have proposed a new approach to the analysis of industrial location patterns that focuses on the presence or absence of industries rather than their percentage distributions across locations. In particular we have found that by choosing MEAs as the appropriate geographic unit, this approach reveals a strong empirical relation between the average size and number of MEAs in which each industry is present, designated here as the NAS Rule. This rule appears to have certain implications for regional development policies as well as for regional economic theory in general. We address these issues in turn, and close with a brief discussion of certain directions for further research.

6.1 Some implications for regional development

One important regional policy objective is to identify those industries that are potentially sustainable in a given region (MEA). Our analysis suggests that such sustainability depends not only on region-specific factors,[51] but also the global structure of the regional system. Such global considerations are often ignored in regional industrial policy decisions. For example, several peripheral cities in Japan recently attempted to attract new IT industries to boost their economy, without much success. Such industrial policies are motivated mainly by the fact that these industries are currently growing the fastest. But IT industries are by nature *high-order* industries, i.e., are typically found in large MEAs. Our findings suggest that there may in fact be little freedom in the location pattern of industries, and in particular that there is a stable relationship between the number and size of MEAs in which a given industry can successfully locate. With respect to size in particular, the Hierarchy Principle suggests that there is the *critical MEA size* for each industry, and that only MEAs with sizes greater than this level can provide viable locations for that industry. Hence to attract such high-order industries, it would appear that these cities should focus first on attracting those lower-order (feasible) industries that will most help to stimulate regional growth. More generally, knowledge of prevailing global industrial location patterns should enhance the efficiency of regional industrial development policies.

6.2 Some implications for regional economic theory

One implication of the NAS Rule for regional economic theory is to underscore the need for more spatially disaggregated models. Most theoretical analyses of economic location are based on simple, highly aggregated models. Perhaps the most popular spatial simplification is the widely used two-region model. For example, many models of economic agglomeration have been developed within this setting.[52] Such models are often a good starting point for analysis in that they tend to allow the simplest possible formalizations

[51] An example is the so-called first nature advantage of certain regions, such as the presence of natural harbors or oil fields. Regional factor endowments can also be a crucial determinant of industrial sustainability when production factors are immobile across regions, which is quite likely in both international and short-term analyses.

[52] See, e.g., Fujita and Thisse [19] for a survey.

of spatial economic behavior. However, when it comes to explaining the actual spatial distribution of industries, say at the national level, such models are generally not very useful. In particular, they are of little help in formulating practical regional industrial development policies in that they do not allow explicit quantitative evaluations of the potential viability of given industries in given cities or regions. In addition, these simplified models often yield drastically different spatial equilibrium configurations (e.g., complete concentration of industries in one region, or complete uniformity of industries across regions) depending on parameter choices. But our empirical results suggest that in reality there may be greater spatial regularity than is implied by these models. Indeed it would appear that too much aggregation of both regions and industries can sometimes obscure the actual regularities embodied in industrial location patterns. Our results thus suggest that a certain degree of model complexity may (paradoxically) be essential to reveal the simple structure of actual spatial economies.

A second implication relates to the theoretical relations established between the Rank-Size Rule, the Hierarchy Principle, and the NAS Rule (Theorem 1 and Corollary 4). In particular, the first two of these regularities have both been studied extensively from theoretical viewpoints. Efforts to account theoretically for the Rank-Size Rule go back at least to the work of Simon [48], who derived this rule as the steady state of a simple random city-growth process. This literature has been recently revitalized by Krugman [31], and by Gabaix [21] (see also a survey by Gabaix and Ioannides [22]). Certain more economically based steady-state approaches have also been developed, as for example in Eeckhout [16], Duranton [10], Rossi-Hansberg and Wright [47], and Córdoba [6]. With respect to the Hierarchy Principle, theoretical interest has focused mainly on the role of demand externalities in the determination of industrial location. In a variety of spatial modeling contexts, such externalities can be shown to lead to co-agglomerations of different industries in equilibrium, as for example in the "lock-in effect" of industrial locations suggested by some of the new economic geography models (e.g., Fujita, Krugman, and Mori [20] and Tabuchi and Thisse [50].)

But to our knowledge, these two streams of research have thus far been quite independent. On the one hand, random growth approaches to the Rank Size Rule typically assume that all mobile industries are symmetric in terms of their location behavior (i.e., no industries are more localized or ubiquitous than others). Moreover, there are no explicit incentives for co-localization of different industries. Hence in these random growth processes there are no natural mechanisms that might lead to the Hierarchy Principle. On the other hand, while the Hierarchy Principle does hold for many of the new economic geography models, there has so far been little attempt to examine the city-size distributions implied by such models. Hence the present results suggest the need for combined approaches that exhibit both types of regularities. In particular, by combining the models developed for explaining the Rank-Size Rule and the Hierarchy Principle, it may be possible to give behavioral explanations for the NAS Rule as well.

Such investigations will be pursued in subsequent work.

6.3 Additional directions for further research

There are at least two possible extensions of the research presented here that should be mentioned. The first is on the appropriate regional level at which the regularities of economic location patterns such as the Hierarchy Principle, the NAS Rule and the Rank-Size Rule should be studied. As noted in the introduction, the monopolarity of the Japanese city system provided a natural starting point for the empirical investigation of these three regularities. But more generally, it is not clear why the Rank-Size Rule should be tested only at the national level. While national boundaries of course constitute natural barriers both in terms of population and industrial location, it may nonetheless be true that certain subsets of regions within a given country represent more meaningful spatial economic systems than the entire country (in the same way that "administrative cities" can be questioned as the relevant geographic units.) This is especially important for countries like the US that are large in terms of both population and area. For the US in particular, it might be more natural to consider, say, three comparably sized regional subsystems (East, West and Midwest) with similarly sized central cities (New York, Los Angeles and Chicago metro areas).

For example, in the same way that commuting patterns are used to define MEAs, one may take the patterns of interregional travel behavior to define relatively cohesive subsystems of regions within national economies. In Japan for example, it appears that based on interregional travel flows there is a natural nesting of three monopolar regional systems, "Tokyo"$\supset$"Osaka"$\supset$"Nagoya", with respective central cities defined by the Tokyo, Osaka and Nagoya metro areas (as discussed in Mori, Nishikimi and Smith [35]). From this viewpoint, it is then natural to ask whether empirical regularities of industrial location patterns are more readily identifiable at these subsystem levels. For Japan in particular, it turns out that the NAS Rule holds with roughly the same slope coefficient for the "Tokyo", "Osaka", and "Nagoya" regions, but is not evident in smaller regional subsystems. Hence it is our view that if international comparisons of these regularities are to be meaningful, they should start with the identification of the most appropriate economic subregions within each country for testing these regularities. Such questions will be pursued further in subsequent research.

Second, while the present analysis has focused on comparisons of location patterns between industries, there may in fact be a wide variation in the locational patterns of functional units within firms (e.g., headquarters, research and development, manufacturing plant, etc.). In fact, informal studies suggest that there is often a positive correlation between the size of a city and the number of cooperate control linkages emanating from that city (e.g., Fujita and Tabuchi [18]; Pred [44]; Ross [46]). However, there have so far been very few formal empirical studies of intra-firm spatial organizations (with the notable exception of Duranton and Puga [15] and Duranton and Overman [13]). Hence, given the central role played by multi-unit firms in modern economies,[53] an important direction for further research is to consider how patterns of spatial organization at the intra-firm level might relate to our present findings at the industry level.

6.4 Toward a more systematic approach to industrial agglomeration

Finally, we consider certain possible refinements of the notion of industry-choice metro areas. Our present approach simply includes all metro areas where the industry is found. But, as mentioned in section 2.2 above, this ignores variations in location density, and thus ignores the spatial agglomerations that are so characteristic of industrial location patterns. Hence to obtain a deeper understanding of the relations between industrial location and city size, it seems essential to us to develop richer statistical models for the analysis of industrial agglomeration structures. As a step in this direction, Mori and Smith [36] develop a simple probability model of multiple agglomerations, and then use a range of standard model-selection criteria to identify a "best" system of agglomerations for each industry. Here, multiple agglomerations are characterized by a simple partition of space into disjoint "agglomeration" region and a residual "non-aggomeration" region, where it is postulated that establishments are more likely to locate in the agglomeration regions. The novelty of this approach is that it treats the entire spatial distribution of agglomerations as a single model, and seeks to determine a optimal model based on information-theoretic criterion.

[53] See, e.g., Duranton and Puga [15], Mun [38], and Ohta and Fujita [41] for recent theoretical analyses of the location of multi-unit firms

7 Appendix

7.1 Location patterns under alternative employment thresholds

Here, we plot the relationships between average size and number of industry-choice MEAs using alternative threshold employment levels to identify the presence of industries in MEAs. In particular, we now say that a given industry is present in an MEA iff the employment share the industry in the MEA is greater than a *threshold fraction*, t, of that in the nation. In the text we have used an implicit threshold fraction, $t = 0$. Figure 7.1 (a,b,c) shows the case for 1999/2000 with $t = 0.1, 0.25$, and 0.5, respectively. Here we can observe that more industries drop from the "log- linear relationship" as the threshold fraction is increased. This is to be expected, since industries with small employment levels in particular are automatically excluded by larger threshold fractions. Note however that the majority of industries still remain around the log linear relationship established for $t = 0$.

7.2 Location patterns by industrial sectors

Figure 7.2 shows the relation between the average size and number of industry-choice MEAs separately for each one-digit industrial sector (manufacturing, services, wholesale and retail) in 1999/2000. Manufacturing is seen to be the most diverse in terms of its location pattern, while services and wholesale-retail are mostly ubiquitous. It is remarkable that essentially the same log-linear relationship is exhibited by all three industrial sectors (except for the two outliers in manufacturing).

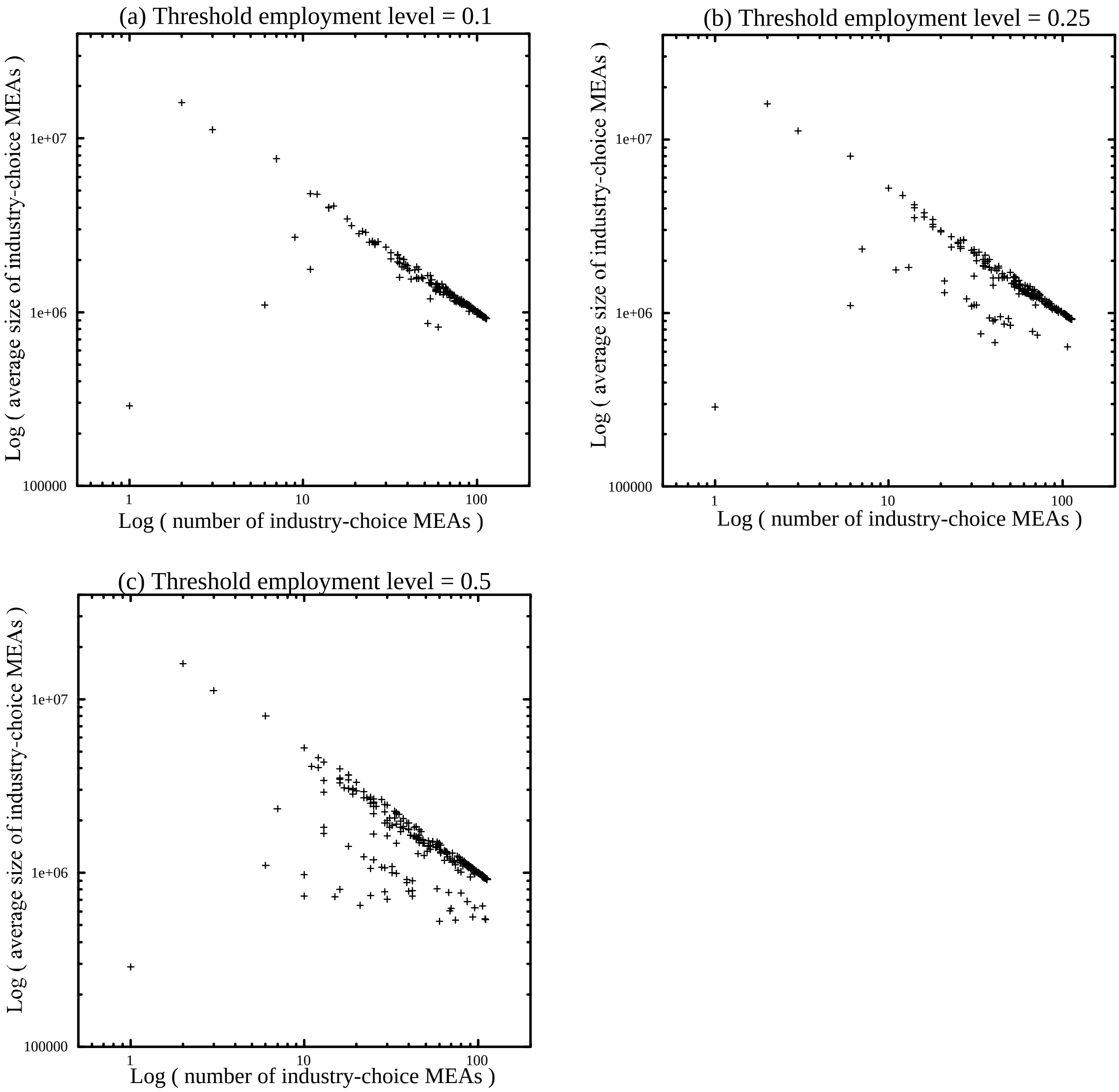

Figure 7.1: Average size of industry-choice MEAs under alternative thresholds for employment size (1999/2000)

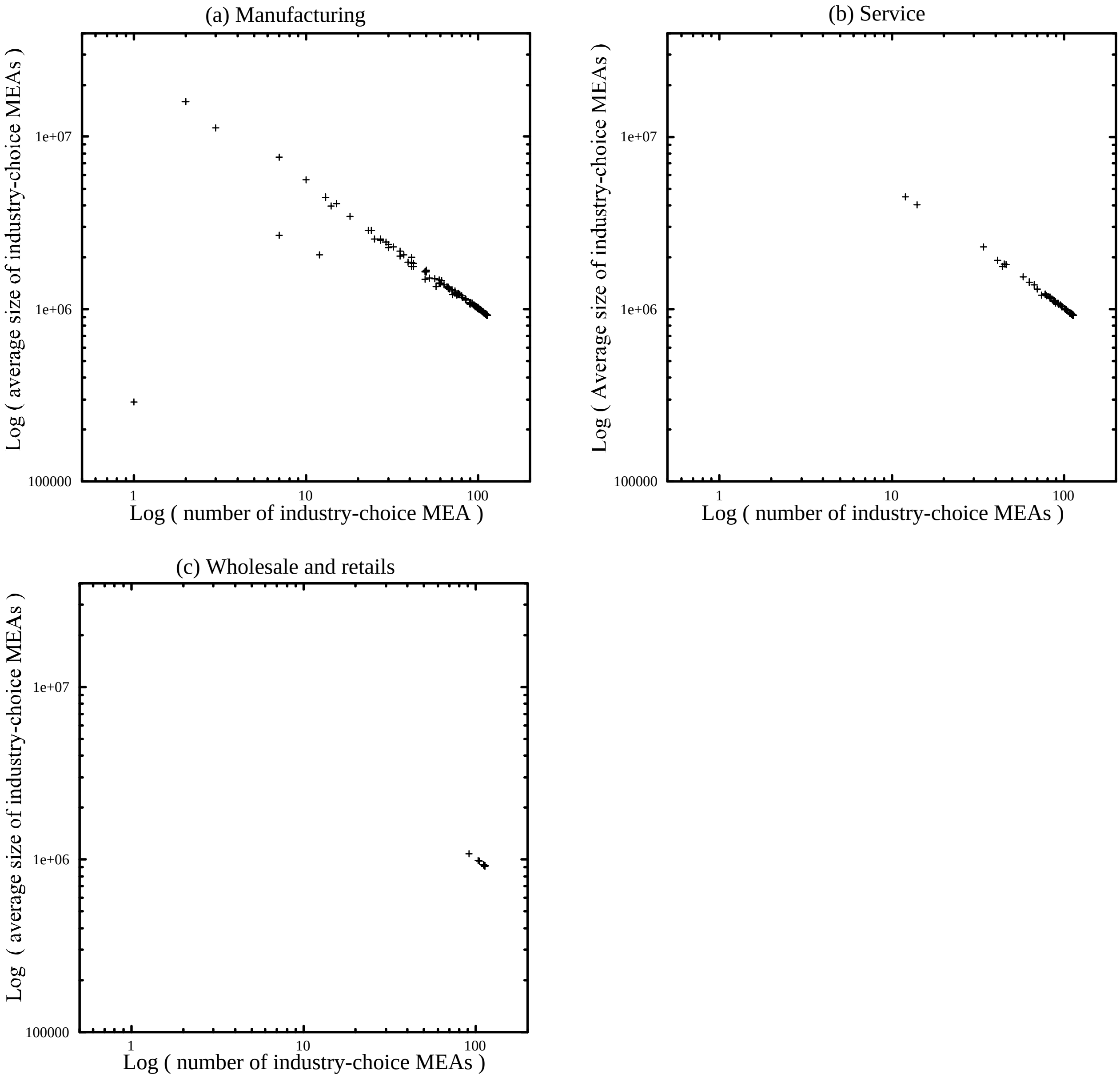

Figure 7.2: Average size of industry-choice MEAs by sectors (1999/2000)

7.3 Formal Analysis of the Discrete Model

The main objective of this section is to prove Theorem 3 in the text. To do so, we begin with a number of preliminary results. The first two are designed to illuminate the properties of condition (5.13) for strong asymptotic β-power laws. We begin by showing that this condition holds identically for the sequence $(\theta_r \equiv 1)$, i.e., that

Proposition 1 *For all $\beta > 0$,*

$$\lim_{r\to\infty} r\left[1-\left(\frac{r}{r-1}\right)^{\beta}\right] = -\beta \tag{7.1}$$

Proof: If the (continuously differentiable) functions f and g are defined respectively for all $x > 0$ by

$$f(x) = 1-\left(\frac{x}{x-1}\right)^{\beta} \tag{7.2}$$

$$g(x) = 1/x \tag{7.3}$$

so that $\lim_{x\to\infty} f(x) = 0 = \lim_{x\to\infty} g(x)$, then it follows by L'Hospital's Rule that

$$\lim_{x\to\infty} x\left[1-\left(\frac{x}{x-1}\right)^{\beta}\right] = \lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{f'(x)}{g'(x)} \tag{7.4}$$

must hold whenever the last limit exists. But

$$f'(x) = -\beta\left[\frac{1}{x-1} - \frac{x}{(x-1)^2}\right] = \frac{\beta}{(x-1)^2} \tag{7.5}$$

$$g'(x) = -\frac{1}{x^2} \tag{7.6}$$

then imply that

$$\lim_{x\to\infty}\frac{f'(x)}{g'(x)} = -\beta\lim_{x\to\infty}\left(\frac{x}{x-1}\right)^2 = -\beta \tag{7.7}$$

and hence that (7.1) must hold. ■

Next we recall that for any convergent sequence, $(\theta_r) \to 1$, the residuals $|1-\theta_r|$ are said to be of *lower order of magnitude* than $1/r$, and written as

$$|1-\theta_r| = o(1/r) \tag{7.8}$$

iff $\lim_{r\to\infty} r\,|1-\theta_r| = 0$. With this definition, we have the following extension of Proposition 1:

Proposition 2 For all $\beta > 0$ and sequences (θ_r) satisfying (7.8),

$$\lim_{r\to\infty} r\left[\theta_r - \left(\frac{r}{r-1}\right)^{\beta}\theta_{r-1}\right] = -\beta \tag{7.9}$$

Proof: First note that for all $r > 1$,

$$r\left[\theta_r - \left(\frac{r}{r-1}\right)^{\beta}\theta_{r-1}\right] = r\left[1 - \left(\frac{r}{r-1}\right)^{\beta}\right] + \left\{r(\theta_r - 1) - \left(\frac{r}{r-1}\right)^{\beta+1}(r-1)(\theta_{r-1} - 1)\right\} \tag{7.10}$$

By taking limits of both sides, we then see from Proposition 1 that it suffices to show that the second term converges to zero. But by hypothesis:

$$\begin{aligned}&\lim_{r\to\infty}\left\{r(\theta_r - 1) - \left(\frac{r}{r-1}\right)^{\beta+1}(r-1)(\theta_{r-1} - 1)\right\}\\ &= \lim_{r\to\infty} r(\theta_r - 1) - \left[\lim_{r\to\infty}\left(\frac{r}{r-1}\right)^{\beta+1}\right]\left[\lim_{r\to\infty}(r-1)(\theta_{r-1} - 1)\right]\\ &= (0) - [1](0) = 0\end{aligned} \tag{7.11}$$

and the result follows. ■

It should be noted that while condition (7.8) is sufficient to guarantee (7.9), it is by no means the most general condition. For example, it should be clear from the argument in (7.11) that it is enough to require that $r(1 - \theta_r)$ have any finite limit.[54]

The next two results are directed toward the proof of Theorem 3. To motivate the first result, observe from (5.7) in the text (with $\alpha = 1$, $\beta \in (0,1)$ and $r = r_i = n_i$) that

$$\begin{aligned}\frac{1}{r}\int_0^r x^{-\beta}dx = \frac{r^{-\beta}}{1-\beta} \quad &\Rightarrow \quad \frac{1}{r^{1-\beta}}\int_0^r x^{-\beta}dx = \frac{1}{1-\beta}\\ &\Rightarrow \quad \lim_{r\to\infty}\frac{1}{r^{1-\beta}}\int_0^r x^{-\beta}dx = \frac{1}{1-\beta}\end{aligned} \tag{7.12}$$

The following result establishes a limiting discrete analogue of this result:

Lemma 3 For all $\beta \in (0,1)$,

$$\lim_{r\to\infty}\frac{1}{r^{1-\beta}}\sum\nolimits_{s=1}^{r} s^{-\beta} = \frac{1}{1-\beta} \tag{7.13}$$

Proof: The result follows from standard upper and lower integral approximations to the summation in (7.13), namely:

$$\int_1^{r+1} x^{-\beta}dx \le \sum\nolimits_{s=1}^{r} s^{-\beta} \le \int_0^r x^{-\beta}dx \tag{7.14}$$

By calculating these integrals we see that

$$\frac{1}{1-\beta}\left[(r+1)^{1-\beta} - 1\right] \le \sum\nolimits_{s=1}^{r} s^{-\beta} \le \frac{1}{1-\beta}r^{1-\beta}$$

[54] However, not even this condition is necessary. To see the difficultly of a general characterization here, it is of interest to note that condition (7.9) can in fact be satisfied by *divergent* sequences (θ_r). Consider for example the divergent sequence, $\left(\theta_r = 1 + n^{\beta}\right)$.

$$\Rightarrow \frac{1}{1-\beta}\left[(\frac{r+1}{r})^{1-\beta} - \frac{1}{r^{1-\beta}}\right] \leq \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta} \leq \frac{1}{1-\beta} \tag{7.15}$$

Hence by taking limits,

$$\frac{1}{1-\beta} = \frac{1}{1-\beta} \lim_{r\to\infty} \left[\left(\frac{r+1}{r}\right)^{1-\beta} - \frac{1}{r^{1-\beta}}\right] \leq \lim_{r\to\infty} \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta} \leq \frac{1}{1-\beta}$$

we obtain the desired result. ■

Next we extend this results to include error factors converging to one:

Lemma 4 For all $\beta \in (0,1)$ and sequences (θ_r) with $\lim_{r\to\infty} \theta_r = 1$,

$$\lim_{r\to\infty} \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta}\theta_s = \frac{1}{1-\beta} \tag{7.16}$$

Proof: If $\lim_{r\to\infty} \theta_r = 1$ then for each $\varepsilon > 0$ there must be some r_ε sufficiently large to ensure that $\theta_r < 1+\varepsilon$ for all $r \geq r_\varepsilon$. Hence for each $r > r_\varepsilon$,

$$\begin{aligned}
\sum\nolimits_{s=1}^{r} s^{-\beta}\theta_s &= \sum\nolimits_{s=1}^{r_\varepsilon - 1} s^{-\beta}\theta_s + \sum\nolimits_{s=r_\varepsilon}^{r} s^{-\beta}\theta_s \\
&\leq \sum\nolimits_{s=1}^{r_\varepsilon - 1} s^{-\beta}\theta_s + (1+\varepsilon)\sum\nolimits_{s=r_\varepsilon}^{r} s^{-\beta} \\
&\leq \sum\nolimits_{s=1}^{r_\varepsilon - 1} s^{-\beta}\theta_s + (1+\varepsilon)\sum\nolimits_{s=1}^{r} s^{-\beta}
\end{aligned}$$

$$\Rightarrow \quad \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta}\theta_s \leq \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r_\varepsilon - 1} s^{-\beta}\theta_s + (1+\varepsilon)\frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta} \tag{7.17}$$

Hence by taking limits in (7.17) we see from Lemma 1 above that:

$$\lim_{r\to\infty} \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta}\theta_s \leq (0) + (1+\varepsilon)\frac{1}{1-\beta} \tag{7.18}$$

The same argument with r_ε large enough to ensure that $\theta_r > 1-\varepsilon$ for all $r \geq r_\varepsilon$ is easily seen to imply that

$$\lim_{r\to\infty} \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta}\theta_s \geq (1-\varepsilon)\frac{1}{1-\beta} \tag{7.19}$$

Finally, since $\varepsilon > 0$ was chosen arbitrarily, the result follows from (7.18) and (7.19). ■

With these preliminary results, we can now establish:

Theorem 3. *For any exponent, $\beta \in (0,1)$, a ranked population sequence N satisfies an asymptotic β-power law with scale factor α iff its upper-average sequence $\widetilde{N}$ satisfies a strong asymptotic β-power law with scale factor $\alpha/(1-\beta)$.*

Proof: To establish this result we first observe that if for any choice of positive scalars α and $\widetilde{\alpha}$ we define the ratio sequences (θ_r) and $(\widetilde{\theta}_r)$ for all $r \in \mathbf{R}$ by

$$\theta_r = \frac{N_r}{\alpha r^{-\beta}} \tag{7.20}$$

$$\widetilde{\theta}_r = \frac{\widetilde{N}_r}{\widetilde{\alpha} r^{-\beta}} \tag{7.21}$$

then without loss of generality we may rewrite (5.10) as

$$\widetilde{\alpha} r^{-\beta} \widetilde{\theta}_r = \frac{1}{r} \sum\nolimits_{s=1}^{r} \alpha s^{-\beta} \theta_s \ , \ r \in \mathbf{R} \tag{7.22}$$

In these terms, it suffices to show that if the scale factors are chosen to satisfy $\widetilde{\alpha} = \alpha/(1-\beta)$, then the ratio sequence (θ_r) satisfies (5.12) iff the ratio sequence $(\widetilde{\theta}_r)$ satisfies (5.12) and (5.13). We begin by showing that if $\widetilde{\alpha} = \alpha/(1-\beta)$ then

$$\lim_{r \to \infty} \theta_r = 1 \ \ \Rightarrow \ \ \lim_{r \to \infty} \widetilde{\theta}_r = 1 \tag{7.23}$$

To do so, simply observe from (7.22) that

$$\widetilde{\theta}_r = \frac{\alpha}{\widetilde{\alpha}} \frac{1}{r^{1-\beta}} \sum\nolimits_{s=1}^{r} s^{-\beta} \theta_s \tag{7.24}$$

and hence from Lemma 2 that

$$\lim_{r \to \infty} \theta_r = 1 \ \ \Rightarrow \ \ \lim_{r \to \infty} \widetilde{\theta}_r = \frac{\alpha}{\widetilde{\alpha}} \left(\frac{1}{1-\beta} \right) = 1 \tag{7.25}$$

Next we consider the sequence of first differences implied by (7.22). In particular, if one multiplies through (7.22) by r to obtain

$$\sum\nolimits_{s=1}^{r} \alpha s^{-\beta} \theta_s = \widetilde{\alpha} r^{1-\beta} \widetilde{\theta}_r \tag{7.26}$$

then subtracting the same expression evaluated at $r-1$ yields the following sequence of relations:

$$\alpha r^{-\beta} \theta_r = \widetilde{\alpha} r^{1-\beta} \widetilde{\theta}_r - \widetilde{\alpha} (r-1)^{1-\beta} \widetilde{\theta}_{r-1} \tag{7.27}$$

Next, dividing through by $\widetilde{\alpha} r^{-\beta}$ to obtain:

$$\begin{aligned} \frac{\alpha}{\widetilde{\alpha}} \theta_r &= r \widetilde{\theta}_r - \left(\frac{r-1}{r} \right)^{-\beta} (r-1) \widetilde{\theta}_{r-1} \\ &= \widetilde{\theta}_r + (r-1) \left[\widetilde{\theta}_r - \left(\frac{r-1}{r} \right)^{-\beta} \widetilde{\theta}_{r-1} \right] \end{aligned} \tag{7.28}$$

It follows that if we again set $\widetilde{\alpha} = \alpha/(1-\beta)$ then by (7.23) and (7.28),

$$\begin{aligned} & \lim_{r \to \infty} \theta_r = 1 \ \ \Rightarrow \ \ \lim_{r \to \infty} \widetilde{\theta}_r = 1 \\ & \Rightarrow \ \ 1 - \beta = 1 + \lim_{r \to \infty} \left\{ (r-1) \left[\widetilde{\theta}_r - \left(\frac{r-1}{r} \right)^{-\beta} \widetilde{\theta}_{r-1} \right] \right\} \\ & \Rightarrow \ \ -\beta = \lim_{r \to \infty} \left(\frac{r-1}{r} \right) \left\{ r \left[\widetilde{\theta}_r - \left(\frac{r-1}{r} \right)^{-\beta} \widetilde{\theta}_{r-1} \right] \right\} \\ & \Rightarrow \ \ -\beta = \lim_{r \to \infty} \left[\widetilde{\theta}_r - \left(\frac{r-1}{r} \right)^{-\beta} \widetilde{\theta}_{r-1} \right] \end{aligned} \tag{7.29}$$

Thus that the the ratio sequence $(\widetilde{\theta}_r)$ also satisfies (5.13), and it may be concluded that $\widetilde{N}$ satisfies a strong asymptotic β-power law with scale factor $\alpha/(1-\beta)$ whenever N satisfies an asymptotic β-power law with scale factor α. Conversely, if the ratio sequence $(\widetilde{\theta}_r)$ satisfies (5.12) and (5.13) with scale factor $\widetilde{\alpha} = \alpha/(1-\beta)$, then again by (7.28):

$$\begin{aligned}(1-\beta)\theta_r &= \widetilde{\theta}_r + (r-1)\left[\widetilde{\theta}_r - \left(\frac{r-1}{r}\right)^{-\beta}\widetilde{\theta}_{r-1}\right] \\ &= \widetilde{\theta}_r + \left(\frac{r-1}{r}\right)\left\{r\left[\widetilde{\theta}_r - \left(\frac{r-1}{r}\right)^{-\beta}\widetilde{\theta}_{r-1}\right]\right\}\end{aligned}$$

$$\begin{aligned}\Rightarrow\quad (1-\beta)\lim_{r\to\infty}\theta_r &= \lim_{r\to\infty}\widetilde{\theta}_r + \lim_{r\to\infty}\left(\frac{r-1}{r}\right)\left\{r\left[\widetilde{\theta}_r - \left(\frac{r-1}{r}\right)^{-\beta}\widetilde{\theta}_{r-1}\right]\right\} \\ &= 1 + \lim_{r\to\infty}\left\{r\left[\widetilde{\theta}_r - \left(\frac{r-1}{r}\right)^{-\beta}\widetilde{\theta}_{r-1}\right]\right\} \\ &= 1-\beta\end{aligned}$$

$$\Rightarrow\quad \lim_{r\to\infty}\theta_r = 1 \tag{7.30}$$

Hence the ratio sequence (θ_r) satisfies (5.12) , and we may conclude that N satisfies an asymptotic β-power law with scale factor α whenever $\widetilde{N}$ satisfies a strong asymptotic β-power law with scale factor $\alpha/(1-\beta)$. ■

Finally, it is of some interest to note that for $\beta > 1$, our definitions do yield certain trivial log linear relationships. For example if N satisfies an exact β-power law, say $N_r = \alpha r^{-\beta}$ with $\beta > 1$ then by noting convergence of the infinite sum:

$$\sum\nolimits_{s=1}^{\infty} s^{-\beta} = c_\beta > 0 \tag{7.31}$$

it follows trivially that the sequence of error factors (θ_r) defined by

$$\theta_r \equiv \frac{\widetilde{N}_r}{\alpha c_\beta r^{-1}} = \frac{r^{-1}\sum_{s=1}^{r}\alpha s^{-\beta}}{\alpha c_\beta r^{-1}} = \frac{1}{c_\beta}\sum\nolimits_{s=1}^{r} s^{-\beta} \tag{7.32}$$

must converge to one. Hence if N satisfies an exact β-power law then its upper average sequence $\widetilde{N}$ must always satisfy an asymptotic β-power law with $\beta = 1$. But since the critical slope coefficient is identically one, such relations appear to have little substance.[55]

[55] However, such relations do imply that if the upper average sequence $\widetilde{N}$ were to satisfy any β-power law (asymptotic or exact) with $\beta > 1$, then the underlying ranked population sequence N could never satisfy a rank-size rule (asymptotic or exact).

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